MATH 1210 Assignment #5

Due: March 30, 2016; At the start of class

Reminder: all assignments *must* be accompanied by a signed copy of the honesty declaration available on the course website.

1. Given that the system has a unique solution, find it using the Cramer's rule:
   a)

$$\begin{aligned} 3x + 5y - 6z &= 8 \\ 7x - z &= 10 \\ 2x + 2y + z &= 2 \end{aligned}$$

   b)

$$\begin{aligned} 30x_1 + 40x_2 + 100x_3 - 143x_4 + x_5 &= 5 \\ 23x_1 - 80x_2 + 46x_3 - 127x_4 + 198x_5 &= -10 \\ 236x_1 + 24x_3 - 27x_4 + 80x_5 &= 0 \\ 123x_1 + 56x_2 - 34x_3 + 56x_5 &= 7 \\ 145x_1 - 64x_2 - 2x_3 + x_4 + 30x_5 &= -8 \end{aligned}$$

   Hint: For part (b), it may not be necessary to calculate 6 5×5 *determinants. Look for simplifications!*

2. Let A and B be square matrices of the same size. Determine if the following statements are always true (justify your answer!):
   a) If A is invertible, then AB is invertible;
   b) If AB is invertible, then A is invertible.

3. Find all values of x for which the matrix $A = \begin{bmatrix} 2x & 5-x & 6 \\ x+3 & x-1 & 3x-3 \\ -40 & 10x^2+30 & 10x+80 \end{bmatrix}$ is singular.

4. a) Using the adjoint method, find the inverse of $A = \begin{bmatrix} 3 & -4 & 5 \\ 6 & 7 & -1 \\ 2 & 8 & 1 \end{bmatrix}$;
   b) Check by definition that the matrix found in (a) is indeed the inverse of A;
   c) Use (a) to solve the system

$$\begin{aligned} 3x - 4y + 5z &= 13 \\ 6x + 7y - z &= 20 \\ 2x + 8y + z &= 23 \end{aligned}$$

d) Use (a) to solve the system

$$\begin{aligned} 3x + 6y + 2z &= 10 \\ -4x + 7y + 8z &= 5 \\ 5x - y + z &= 0 \end{aligned}$$

e) Solve the system $A^{-1}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

5. Let A be a square matrix such that $adjA = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ -1 & -7 & -9 \end{bmatrix}$.

   a) Find $|adjA|$;
   b) Find $|A|$;
   c) Find A^{-1};
   d) Find A.

6. Use the direct method to find inverse of $\begin{bmatrix} 2\sqrt{3} & -1 & -3 \\ 2 & \sqrt{3} & 3\sqrt{3} \\ 0 & -2 & 2 \end{bmatrix}$.

7. Determine if the following vectors are linearly independent or linearly dependent. Justify your answer.
   a) $\langle 1, 2, 4 \rangle, \langle -1, 2, 3 \rangle, \langle 0, 3, -5 \rangle$
   b) $\langle 2, 3, -5, 8 \rangle, \langle 3, 7, 9, 10 \rangle, \langle -4, -6, 10, -16 \rangle$
   c) $\langle 23, 35, 57, 79 \rangle, \langle 23, 34, 45, 56 \rangle, \langle 87, 76, 65, 54 \rangle, \langle 54, 43, 32, 32 \rangle, \langle 35, 50, 75, 23 \rangle$
   d) $\langle -3, 2, 5, 4 \rangle, \langle 3, 7, 8, -10 \rangle, \langle 3, 5, 0, -9 \rangle$
   e) $\langle 1, 2 \rangle, \langle 3, -7 \rangle, \langle 8, 4 \rangle$
   f) $\langle 1, 0, 0, -3 \rangle, \langle 0, 1, 2, 0 \rangle, \langle 1, -3, 0, 0 \rangle, \langle 4, 0, 6, -9 \rangle$

8. Let u, v, w be vectors in 3-dimensional space.
   a) If $w = 3u + 2v$, express $v + w$ as a linear combination of u and w.
   b) Prove that if u, v and w are linearly dependent, then $u - v, v$ and w are linearly dependent.
   c) Is it true that if $u - v, v$ and w are linearly dependent, then u, v and w are linearly dependent? Justify your answer.
   d) Is it true that if $u - v, v - w$ and $w - u$ are linearly dependent, then u, v and w are linearly dependent? Justify your answer.