Definition

The complex number set $\mathbb{C}$ consists of all numbers of the form

$$z = x + iy$$

where x and y are real, and i satisfies $i^2 = -1$.

Definition

Two complex numbers $z_1 = x + yi$ and $z_2 = a + bi$ are equal if their real and imaginary parts are equal:

$$x + yi = a + bi \qquad \Leftrightarrow \qquad x = a \text{ and } y = b.$$

Definition

If $z_1 = x + yi$ and $z_2 = a + bi$ then

$$z_1 + z_2 = (x + a) + (y + b)i$$

$$z_1 - z_2 = (x - a) + (y - b)i$$

Definition

If $z_1 = x + yi$ and $z_2 = a + bi$ then

$$z_1 z_2 = (x + yi)(a + bi) = (xa - yb) + (xb + ya)i$$

Definition

The complex conjugate $\bar{z}$ of the complex number $z = x + iy$ is

$$\bar{z} = x - iy.$$

Theorem

Every real quadratic equation

$$ax^2 + bx + c = 0$$

has

- two real distinct roots if $b^2 - 4ac > 0$,
- two real equal roots if $b^2 - 4ac = 0$,
- two complex conjugate roots if $b^2 - 4ac < 0$.

Solutions x_1 and x_2 take the form of:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$