# Principle of Mathematical Induction (1/2)

Let  $P_n$  be a proposition defined on the set of positive integers  $(\forall n \in \mathbb{N})$ .

#### First Form

If  $P_n$  satisfies the two properties:

- 1.  $P_1$  is a true proposition,
- 2. if  $P_k$  is a true proposition,  $P_{k+1}$  is a true proposition.

Then,  $P_n$  is a true proposition for all positive integers  $(\forall n \in \mathbb{N})$ .

# Principle of Mathematical Induction (2/2)

Let N be an integer.

Let  $P_n$  be a proposition  $\forall n \geq N$ , where  $n \in \mathbb{N}$ .

#### Second Form

If  $P_n$  satisfies the two properties:

- 1.  $P_N$  is a true proposition,
- 2. if  $P_k$  is a true proposition, then  $P_{k+1}$  is a true proposition.

Then,  $P_n$  is a true proposition for all  $n \geq N$ , where  $n \in \mathbb{N}$ .

### Proof technique: Mathematical Induction

To prove that a proposition  $P_n$  is true for all  $n \geq N$ , where  $n \in \mathbb{N}$ .

- 1. Prove that the proposition  $P_n$  is true for the starting value N (usually N = 1 or N = 2).
- 2. Assume that the proposition  $P_n$  is true when n = k. And then prove that it is also true when n = k + 1. In proving the proposition when n = k + 1, use the assumption that  $P_n$  is true when n = k.
- 3. **Conclusion:** Then, by the Principle of Mathematical Induction, we can conclude that  $P_n$  is a true proposition for all  $n \geq N$ , where  $n \in \mathbb{N}$ .