Definition (Linear combination)

Let $\mathbf{v}$ and $\mathbf{u}_1, \ldots, \mathbf{u}_k$ be vectors in n-space. We say that $\mathbf{v}$ is linear combination of the vectors $\mathbf{u}_1, \ldots, \mathbf{u}_k$ if there exists k scalars $c_1, \ldots, c_k$ such that

$$\mathbf{v} = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + \cdots + c_k\mathbf{u}_k.$$

Definition

A set $\{\mathbf{u}_1, \ldots, \mathbf{u}_k\}$ of k nonzero vectors is said to be *linearly independent* if, for $c_1, \ldots, c_k$ scalars,

$$c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + \cdots + c_k\mathbf{u}_k = 0 \Leftrightarrow c_1 = c_2 = \cdots = c_k = 0.$$

If $\{\mathbf{u}_1, \ldots, \mathbf{u}_k\}$ is not linearly independent, we say it is *linearly dependent*.

(Linear dependence of $\{\mathbf{u}_1, \ldots, \mathbf{u}_k\}$ means that (at least) one of the $\mathbf{u}_i$ can be written in terms of some of the others.)

Theorem

Let $\{\mathbf{u}_1, \ldots, \mathbf{u}_m\}$ *be a set of* m *vectors in* n*-space (i.e., each vector has* n *components). Then*

- *If* $m > n$, *then* $\{\mathbf{u}_1, \ldots, \mathbf{u}_m\}$ *is linearly dependent.*
- *If* $m = n$, *form the matrix* $M = [\mathbf{u}_1 \cdots \mathbf{u}_n]$ *by using the* $\mathbf{u}_i$*'s as columns of* M. *Then* $\{\mathbf{u}_1, \ldots, \mathbf{u}_m\}$ *is linearly independent if and only if* $|M| \neq 0$.