Definition

The **modulus** of a complex number $z = x + iy$ is

$$r = |z| = \sqrt{x^2 + y^2}.$$

Definition

The **argument** of a complex number $z = x + iy$, often denoted by θ or *arg* z, is the angle of the rotation of the positive real axis in a counterclockwise direction to the line segment joining $z = 0$ and $z = x + iy$. Clockwise rotations are regarded as negative.

Definition

The value of the argument in the interval $-\pi < \theta \leq \pi$ is called the **principal value** of the argument.

Definition (Polar representation)

A complex number z can be written in the form

$$z = r(\cos\theta + i\sin\theta)$$

where r is the modulus of z and θ is the argument of z.

Compound angle formulas

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

- The modulus of the product of two complex numbers z_1 and z_2 is the product of their moduli:

$$|z_1 z_2| = |z_1||z_2|.$$

- An argument of the product of two complex numbers is the sum of their arguments:

$$arg(z_1 z_2) = arg\, z_1 + arg\, z_2.$$

- The modulus of the quotient of two complex numbers is the quotient of their moduli:

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}.$$

- An argument of the quotient of two complex numbers is the difference of their arguments:

$$arg(\frac{z_1}{z_2}) = arg\, z_1 - arg\, z_2.$$

DeMoivre's Theorem

$$z^n = r^n(\cos n\theta + i \sin n\theta)$$

Euler's Identity

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Definition

The exponential form of a complex number is

$$z = re^{i\theta}$$