Linear Equations

What makes an equation linear?

$$3x + 4y = 2(x + y) + 7$$

- Variables
- Expression
- terms
- coefficient

Equations in one variable, solve for x

$x + 4 = 7$

$3x = 9$

Equations in one variable, solve for x

$$x + \frac{2}{3} = \frac{4}{7}$$

Equations in one variable, solve for x

$$\frac{3}{5}x = \frac{4}{9}$$

Equations in one variable, solve for x

$$5x + 4 = 4x + 3$$

Equations in one variable, solve for x

$$7(x+3) = 2x - 8$$

Equations in one variable, solve for x

$$\frac{1}{3}x + \frac{4}{5} = \frac{1}{2}x - \frac{3}{2}$$

Equations in one variable, solve for x

$$\frac{x}{5} + 4 = 30 - \frac{2x}{3}$$

Equations in one variable, solve for x

$$2(x+5)-4(x+2)=8x+12$$

Equations in one variable, solve for x

$$3(x-2)-4(x+1)=7-x$$

Equations in one variable, solve for x

$$2(4-3x)+5(x+1)=11+x-2(x-1)$$

Linear Inequalities

Definition

A *linear inequality* in x is an inequality that can be expressed in the form

$$ax \leq b\,,$$

where a and b are constants where $a \neq 0$.

A linear inequality may also be expressed as one of the following:

- $ax < b$
- $ax \geq b$
- $ax > b$

Inequalities in one variable, addition and subtraction

$x + 2 \geq 5$

$x - 7 < 12$

Inequalities in one variable, multiplication and division

(Case 1: positive numbers)

$3x \leq 9$

$\frac{1}{2}x > 7$

Inequalities in one variable, multiplication and division

(Case 2: negative numbers)

$-3x \leq 9$

$-\frac{1}{2}x > 7$

Inequalities in one variable, solve for x

$$4x + 5 \geq 6x + 13$$

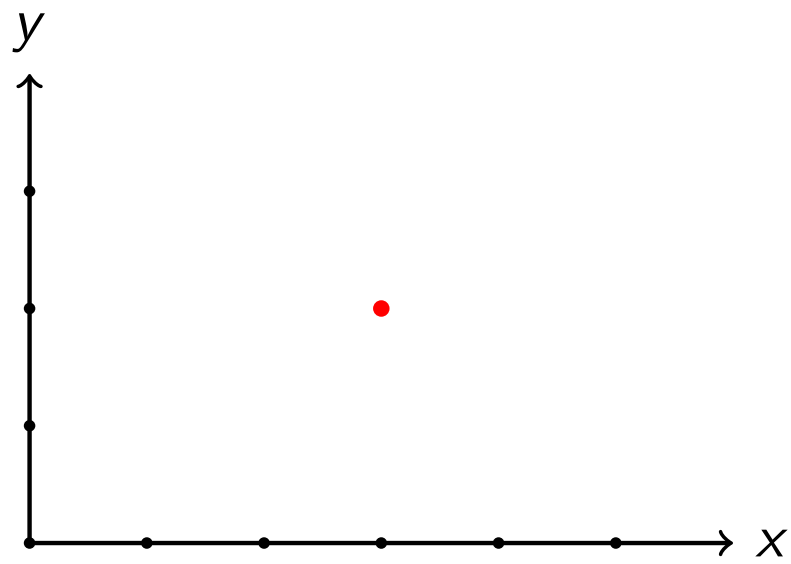
Inequalities in one variable, solve for x

$$3(2y-1)-2(4y-3) \geq 6y+2+5(1-y)$$

Inequalities in one variable, solve for x

$$\frac{1}{2}(x-3)+5<\frac{1}{3}(2x+4)$$

Cartesian coordinates



Linear Equations in two variables

Definition

A *linear equation* in two variables x and y is an equation that an be expressed in the form $Ax + By = C$, where A,B, and C are numbers and A and B are not both zero.

This form is called the *general equation* of the line.

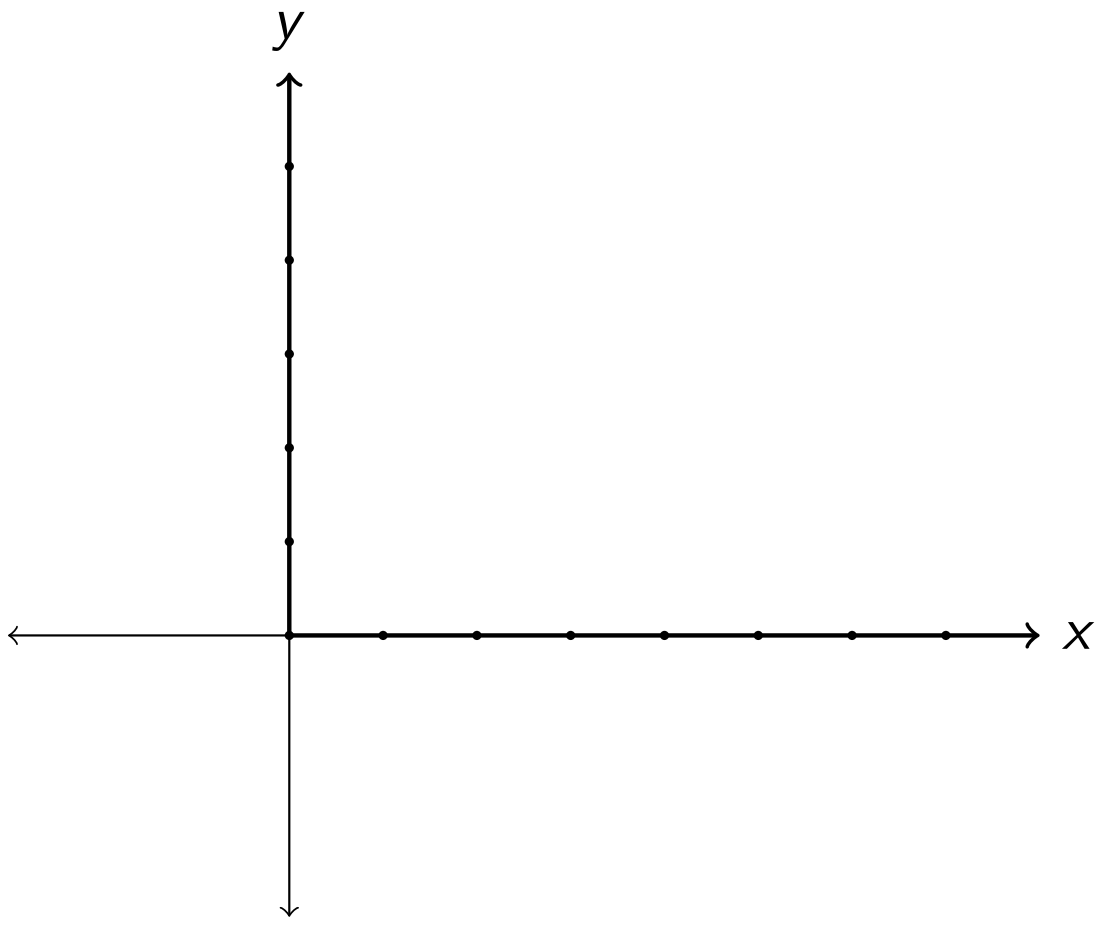
Example: $4x + 3y = 12$

What point is on the line that satisfies $x = 0$?

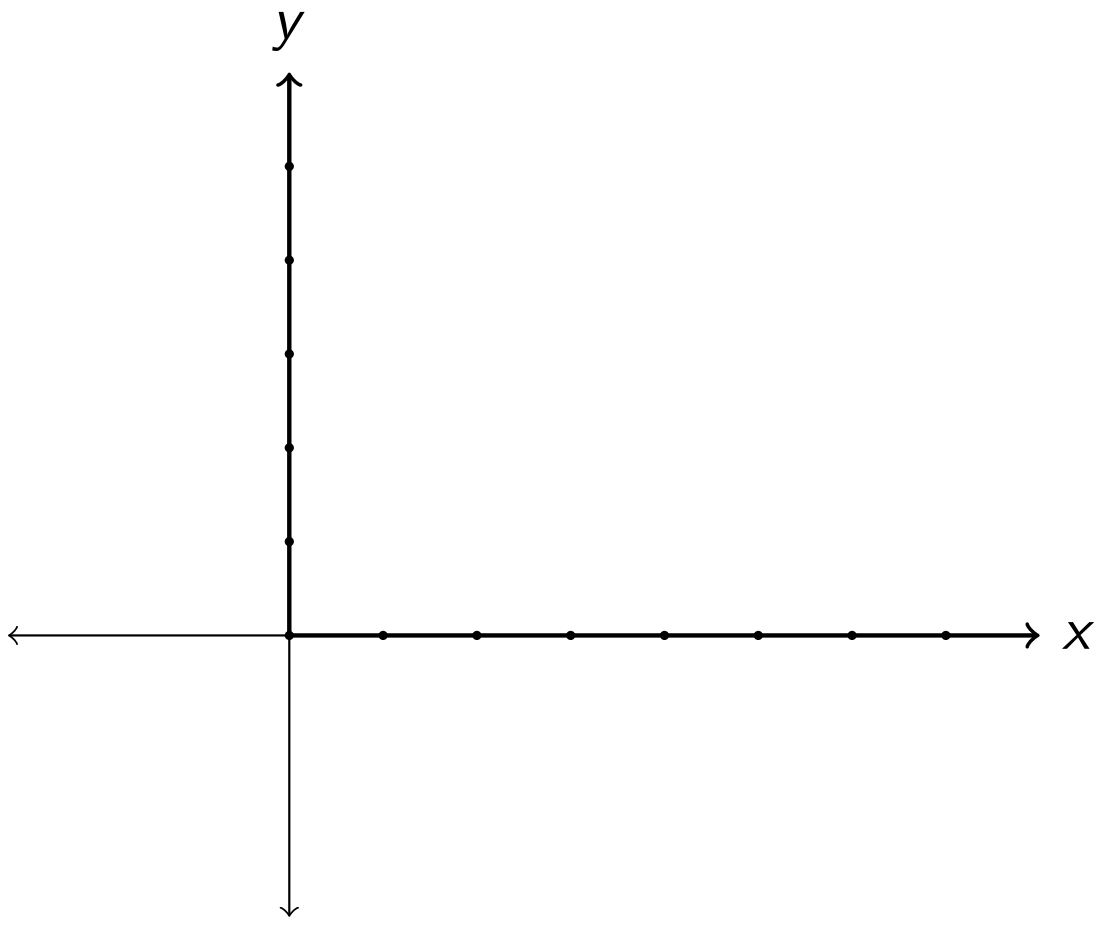
$x = 5$?

$y = 1$?

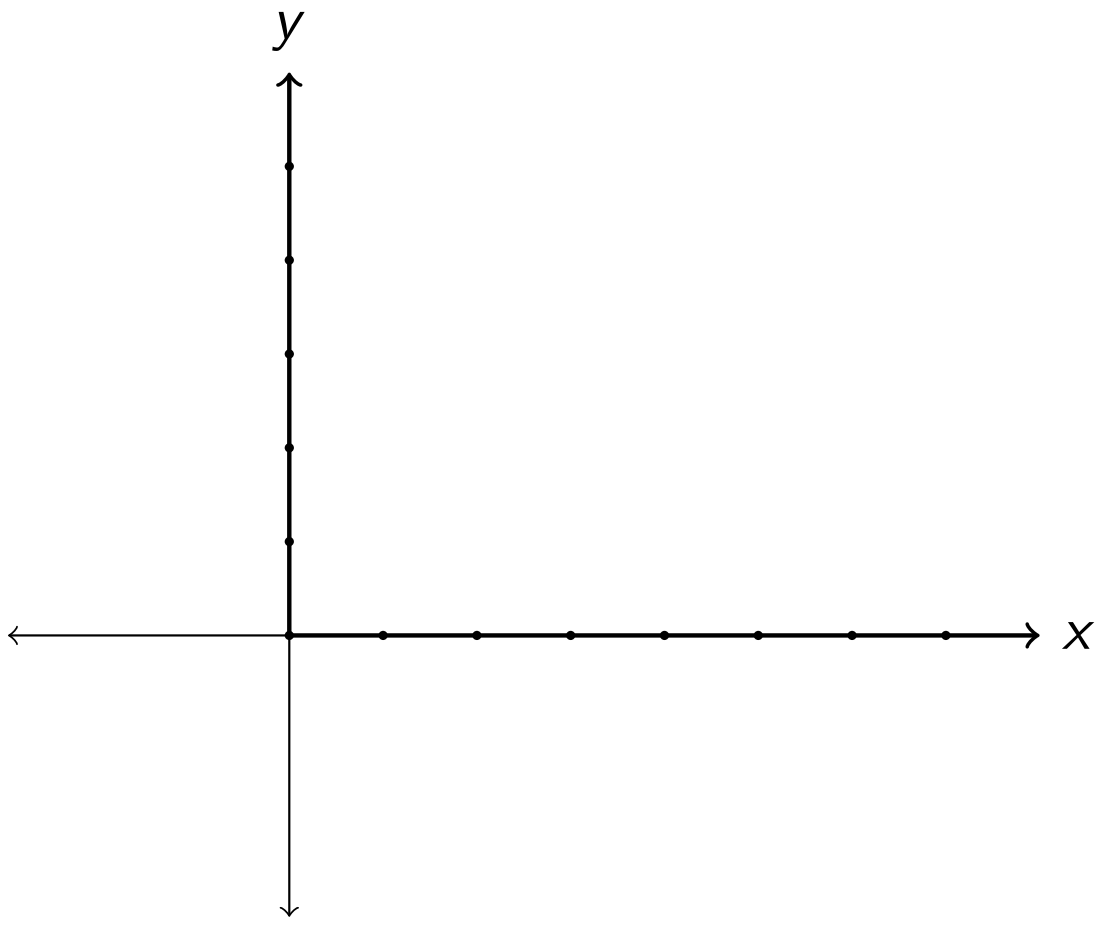
Graph $2x + 7y = 14$



Graph $y = 3$



Graph $2x + 3y = 0$



Line passing through points $(3, 1)$ and $(0, 7)$

Slope

Definition

A line that passes through point (x_1, y_1) and (x_2, y_2) has a *slope* of m defined by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Horizontal lines have a slope of 0.
Vertical lines have an undefined slope (or a slope of ∞).

Equations of Lines

Definition

A linear equation written as $y - y_1 = m(x - x_1)$ is said to be in *point slope* form.

Definition

A linear equation written as $y = mx + b$ is said to be in *slope y-intercept* form.

Line passing through points $(3, 1)$ and $(0, 7)$

Line passing through points $(3, 1)$ and $(2, 1)$

Line passing through points $(4, 2)$ and $(4, 3)$

Example

Find the slope y-intercept form of the line through $(0, 12)$ and $(6, 21)$.

Example

Find a general form of the line through $(-1, 5)$ having slope $\frac{4}{7}$.

Intersection of two lines

Graph the lines $y = x - 1$ and $3x + 2y = 7$.

Intersection of two lines (substitution)

Find the point of intersection the lines $y = x - 1$ and $3x + 2y = 7$.

Intersection of two lines (elimination)

Find the intersection the lines $3x - y = 4$ and $2x + 3y = 7$.

Intersection of two lines

Find the intersection the lines $y = 2x + 1$ and $x + 4y = 20$.

Intersection of two lines

Find the intersection the lines $y = \frac{-2}{3}x + 2$ and $2x + 3y = 6$.

Intersection of two lines

Find the intersection the lines $y = \frac{5}{2}x - 3$ and $5x - 2y = 7$.

parallel lines

Definition

Two distinct lines are *parallel* if they have no points in common.

Theorem

Two distinct lines are parallel if and only if they have the same slope.

parallel lines example

Show the lines $y = \frac{-3}{4} + 2$ and $3x + 4y = 10$ are parallel.

perpendicular lines

Definition

Two distinct lines are *perpendicular* if they intersect at right angles.

Theorem

Two distinct lines with non-zero slope are perpendicular if and only if

$$m_1 = \frac{-1}{m_2}$$

$$(\ ie.\ \ m_1 m_2 = -1)$$

Are they Parallel? Perpendicular? Neither?

line 1	line 2	conclusion
$y = 3x - 3$	$y = 3x - \frac{1}{7}$	
$y = 5x + 7$	$y = -5x - 7$	
$y = \frac{-1}{3}x - \frac{3}{7}$	$y = 3x + 7$	
$y = 4x + 2$	$y = \frac{1}{4}x - \frac{1}{2}$	

Are they Parallel? Perpendicular? Neither?

line 1	line 2	conclusion
$\frac{-1}{3}x + y = 2$	$x + 3y = 2$	
$x + 2y = 7$	$3x + 6y = 7$	
$4x + 3y = 7$	$4y - 3x = 12$	
$3x + 2y = 7$	$6x + 4y = 14$	

Example

Find an equation of a line that passes through the point $(4, -3)$ and is parallel to the line $4x + 3y = 7$.

Example

Find an equation of a line that passes through the point $(4, -3)$ and is perpendicular to the line $4x + 3y = 7$.

Linear Inequalities in Two variables

Definition

A *linear inequality* in two variables x and y is an inequality that can be expressed in the form

$$Ax + By \leq C$$

where A, B, and C are numbers with A and B not both zero. The inequality symbol $\leq$ could be replaced by $<$, $\geq$ or $>$.

$4x - 3y \leq 12$

point	Satisfies Inequality?
$(0, 0)$	
$(2, -2)$	
$(3, 7)$	
$(-1, -5)$	

$4x - 3y \leq 12$

$2x + 5y < 10$

Systems of Linear Inequalities

Definition

The collection of all points that satisfy a number of inequalities is called a *feasible set*. A point in the feasible set is called a feasible point.

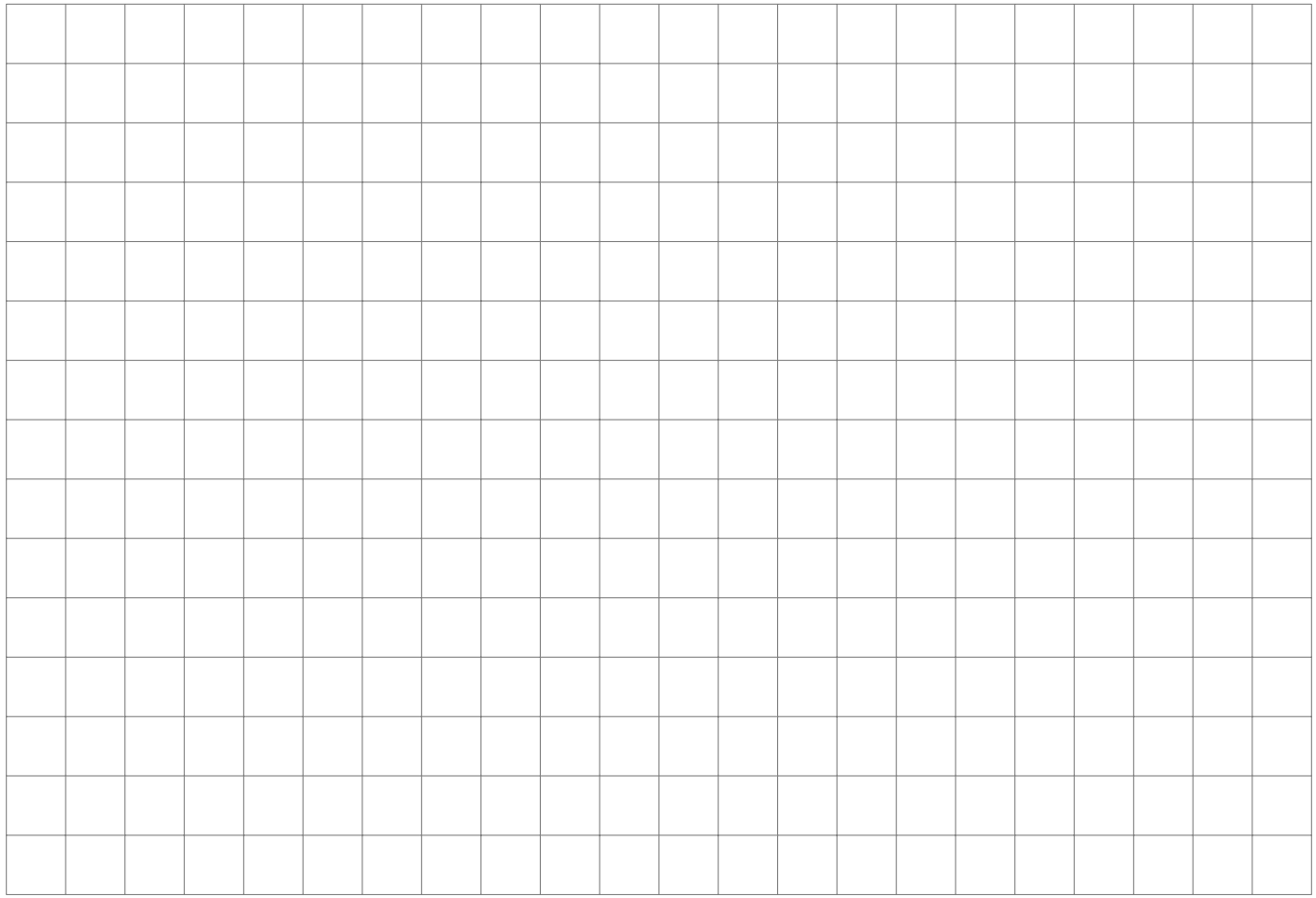
When we graph all of the inequalities in a system, the remaining unshaded points form the feasible set.

Graph the feasible set of the following system:

$4x - 3y \leq 12$

$2x + 5y < 10$

$y \geq 0$



Graph the feasible set of the following system:

$4x + 3y \leq 6$

$x + y < 3$

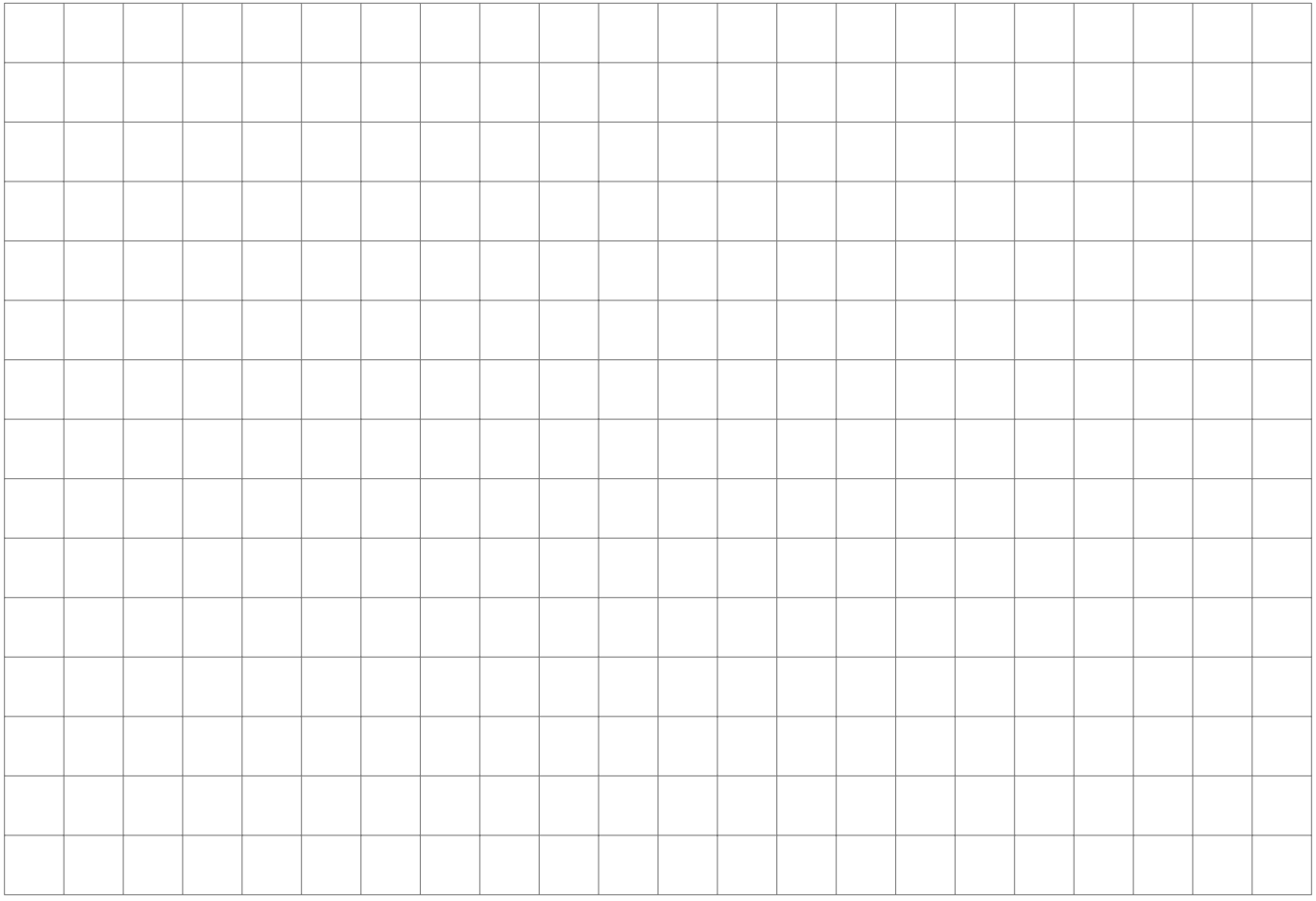
$x \geq 0$

Graph the feasible set of the following system (label all corners):

$3x - 2y \geq 1$

$5x + 2y \leq 23$

$x + 2y \geq 3$



Application Example

Jack and Dianne both sell cars. Jack makes $800 a week plus $10 commission for each car sold. Dianne makes $500 a week plus $25 commission for each car sold.

- Find a linear equation for Jack's weekly income (y) in terms of the number of cars sold (x).
- Find a linear equation for Dianne's weekly income (y) in terms of the number of cars sold (x).
- Find and interpret the y-intercepts of these lines.
- For what number of cars do Jack and Dianne receive the same income? What is that income?

Application Example

The Nifty Nut company makes two kinds of GORP. Plenty Peanuts is $\frac{1}{2}$ peanuts and $\frac{1}{2}$ raisins. Raisin D'etre is $\frac{2}{5}$ peanuts and $\frac{3}{5}$ raisins. They currently have 220kg of peanuts and 280kg of raisins in stock.

- How much of each type of GORP should they make to use all of their supply?

Support your answer. Be sure to identify the variables used.

Application example

In Canada, the consumptions of Fluffernutters has decreased steadily since 1990, as shown by the equation

$$y = -\frac{1}{10}x + 24$$

where y is the amount of Fluffernutter consumed (in millions of kg's), and x is the number of years since 1990.

- What interpertation can be given to the x-intercept?
- What interpertation can be given to the y-intercept?
- When will the consumption be 17 million kg's?

Application example

The following data has been collected on the time it takes to register participants in an online game.

Number of participants	time to completion (in seconds)
2	25
3	27
5	31
7	35

- Show that there is a linear relationship between the time to completion (y) and the number of participants (x).
- What is the slope and what does it represent?
- Find and interpret the y-intercept.

Application Example

Two companies are being considered for water delivery to a small office. Company A charges $30 for delivery and $5 for each 10 litre jug. Company B charges $50 for delivery and $3 for each 10 litre jug.

- For each company, find a linear equation for the total cost (y) of the delivery of x jugs of water.
- If the office needs 9 jugs of water a week, which is the best company for them?
- What number of jugs would cost the same from both companies? What would that cost be?