# Loans and Mortgages - Example (finding Y)

Taylor has a \$2000 loan held at 8.64% which he wants paid off in two years by making payments every two months. What should his payments be?

$$P\left(1+\frac{i}{100m}\right)^n = \frac{Y\left[\left(1+\frac{i}{100m}\right)^n - 1\right]}{\left(\frac{i}{100m}\right)}$$

$P = 2000$

$Y = ?$

$i = 8.64$

$n = 2 \times m = 12$

$m = 6$

$$2000\left(1+\frac{8.64}{600}\right)^{12} = \frac{Y\left[\left(1+\frac{8.64}{600}\right)^{12} - 1\right]}{\left(\frac{8.64}{600}\right)}$$

$$\frac{2000\left(1+\frac{8.64}{600}\right)^{12}\left(\frac{8.64}{600}\right)}{\left[\left(1+\frac{8.64}{600}\right)^{12} - 1\right]} = Y$$

$$182.67536 = Y$$

Round up

Taylor will make payments of \$182.68 every other month.

# Example time permitting

Pat and Kennedy just bought a new home, with a mortgage of \$375000 at 3.25%. They want the mortgage paid off in 20 years. What monthly payment should they be making? (This is a "find Y" question.)

$P = 375\,000$

$Y = ?$

$i = 3.25$

$n = 20 \times m = 240$

$m = 12$

$$P\left(1+\frac{i}{100m}\right)^n = \frac{Y\left[\left(1+\frac{i}{100m}\right)^n - 1\right]}{\left(\frac{i}{100m}\right)}$$

$$375\,000\left(1+\frac{3.25}{1200}\right)^{240} = \frac{Y\left[\left(1+\frac{3.25}{1200}\right)^{240} - 1\right]}{\left(\frac{3.25}{1200}\right)}$$

$$Y = \frac{375\,000\left(1+\frac{3.25}{1200}\right)^{240}\left(\frac{3.25}{1200}\right)}{\left[\left(1+\frac{3.25}{1200}\right)^{240} - 1\right]}$$

$$Y = 2126.984106$$

They should make monthly payments of \$2126.99

# Loans and Mortgages - Example (finding n)

Serge has a loan of \$65000, at an interest rate of 7.95%. He makes payments of \$575 every month. How long before his loan is paid off?

$P = 65000$

$Y = 575$

$i = 7.95$

$n = ?$

$m = 12$

$$P\left(1+\frac{i}{100m}\right)^n = \frac{Y\left[\left(1+\frac{i}{100m}\right)^n - 1\right]}{\left(\frac{i}{100m}\right)}$$

$$65000\left(1+\frac{7.95}{1200}\right)^n = \frac{575\left[\left(1+\frac{7.95}{1200}\right)^n - 1\right]}{\left(\frac{7.95}{1200}\right)}$$

$$430.625\left(1+\frac{7.95}{1200}\right)^n = 575\left(1+\frac{7.95}{1200}\right)^n - 575$$

$$575 = (575 - 430.625)\left(1+\frac{7.95}{1200}\right)^n$$

$$575 = 144.375\left(1+\frac{7.95}{1200}\right)^n$$

$$3.982683983 = \left(1+\frac{7.95}{1200}\right)^n$$

$$\log(3.982683983) = \log\left(1+\frac{7.95}{1200}\right)^n$$

$$\log(3.982683983) = n\log\left(1+\frac{7.95}{1200}\right)$$

$$n = \frac{\log(3.982683983)}{\log\left(1+\frac{7.95}{1200}\right)} = 209.287$$

The loan will be paid off after 210 months (17 years and 6 months).

# Example time permitting

Kelly has a small business loan of \$200 000, held at a rate of 6.3%. Payments are due quarterly. Kelly is able to make payments of \$15 000 each quarter. How long before the loan is paid in full?

(This is a 'find n' question.)

$$P\left(1+\frac{i}{100m}\right)^n = \frac{Y\left[\left(1+\frac{i}{100m}\right)^n - 1\right]}{\left(\frac{i}{100m}\right)}$$

$P = 200\,000$

$Y = 15\,000$

$i = 6.3$

$n = ?$

$m = 4$

note, answer we find will be in terms of quarter years.

$$200\,000\left(1+\frac{6.3}{400}\right)^n = \frac{15000\left[\left(1+\frac{6.3}{400}\right)^n - 1\right]}{\left(\frac{6.3}{400}\right)}$$

$$200\,000\left(1+\frac{6.3}{400}\right)^n = 952\,380.9524\left[\left(1+\frac{6.3}{400}\right)^n - 1\right]$$

$$200\,000\left(1+\frac{6.3}{400}\right)^n = 952\,380.9524\left(1+\frac{6.3}{400}\right)^n - 952\,380.9524$$

$$952\,380.9524 = 752\,380.9524\left(1+\frac{6.3}{400}\right)^n$$

$$1.265822785 = (1.01575)^n$$

$$\log(1.265822785) = \log(1.01575)^n$$

$$\log(1.265822785) = n\log(1.01575)$$

$$n = \frac{\log(1.265822785)}{\log(1.01575)} = 15.084$$

The loan will be fully paid after 16 quarter (or after 4 years)

# Loans and Mortgages - Example (finding PR)

Albert has a loan of \$20000, which has an interest rate of 13.5%. He has been making monthly payments of \$395 for the last 5 years. How much does he still owe?

$$P.R. = P\left(1+\frac{i}{100m}\right)^n - \frac{Y\left[\left(1+\frac{i}{100m}\right)^n - 1\right]}{\left(\frac{i}{100m}\right)}$$

$P = 20000$

$Y = 395$

$i = 13.5$

$n = 5 \times m = 60$

$m = 12$

$$PR = 20000\left(1+\frac{13.5}{1200}\right)^{60} - \frac{395\left[\left(1+\frac{13.5}{1200}\right)^{60} - 1\right]}{\left(\frac{13.5}{1200}\right)}$$

$$= 5544.0284\ldots$$

This expression is large & difficult to correctly punch into a calculator. It may be best to calculate the two parts seperately.

$$PR = 39\,132.90358 - 33\,588.87518$$

$$= 5544.0284$$

after 5 years, \$5544.03 is still owing.

# Example time permitting

Jamie borrowed $3000 from a credit store that charges 19.5% interest. Monthly payments of $250 have been made for a year. How much of the loan is still owing? (This is a "Find PR" question.)

$P = 3000$

$Y = 250$

$i = 19.5$

$n = 1 \times m = 12$

$m = 12$

$$PR = P\left(1 + \frac{i}{100m}\right)^n - \frac{Y\left[\left(1 + \frac{i}{100m}\right)^n - 1\right]}{\left(\frac{i}{100m}\right)}$$

$$= 3000\left(1 + \frac{19.5}{1200}\right)^{12} - \frac{250\left[\left(1 + \frac{19.5}{1200}\right)^{12} - 1\right]}{\left(\frac{19.5}{1200}\right)}$$

$$= 3640.2227 - 3283.1935$$

$$= 357.0292$$

There will be \$357.03 owing after the year.