Preface

This text evolved, as have so many others, from notes used to teach partial differential equations to advanced undergraduate mathematics and physics students and graduate engineering students. Major emphasis is placed on techniques for solving partial differential equations found in physics and engineering, but discussions on existence and uniqueness of solutions are also included. Every opportunity is taken to show that there may be more than one way to solve a particular problem and to discuss the advantages of each solution relative to the others. In addition, physical interpretations of mathematical solutions are stressed whenever possible.

In Chapter 1, we use the method of characteristics to solve first-order quasilinear and general nonlinear equations. Applications included are the one-dimensional wave equation, the eikonal equation from geometric optics, and traffic flow problems.

Section 2.1 introduces second-order equations and describes how initial boundary value problems are associated with such equations. To distinguish between physical assumptions leading to various models of heat conduction, vibration, and potential problems, and the mathematical techniques to solve these problems, models are developed in Sections 2.2–2.6 with no attempt at solutions. At this stage, the reader concentrates only on how mathematics describes physical phenomena. Once these ideas are firmly entrenched, it is then reasonable to proceed to various solution techniques. It has been our experience that confusion often arises when new mathematical techniques are prematurely applied to unfamiliar problems. In this chapter, we also classify second-order PDEs in two variables as being hyperbolic, parabolic, or elliptic, with the wave equation, the heat conduction equation, and Laplace's equation being their canonical forms. The wave equation, together with d'Alembert's solution and its extension to nonhomogenoues problems, is given special consideration. We are careful to point out, however, that such representations of solutions of initial boundary value problems are not possible for parabolic and elliptic equations.

One of the most fundamental classical techniques for solving partial differential equations is that of separation of variables, which leads, in the simplest of examples to trigonometric Fourier series. Chapter 3 develops the theory of Fourier series to the point where it is easily accessible to separation of variables in Chapter 4. The method of *variation of constants* is introduced in order to deal with nonhomogeneities. The examples in Chapter 4 also suggest the possibility of expansions other than trigonometric Fourier series, and these are discussed in detail through Sturm-Liouville systems in Chapter 5. They are then used in Chapter 6 to solve homogeneous problems in one, two, and three space variables. In this chapter, we also illustrate how to verify series solutions of initial boundary value problems, and we discuss distinguishing properties of parabolic, elliptic, and hyperbolic partial differential equations. In Chapter 7, finite Fourier transforms are presented as an alternative to variation of constants for nonhomogeneous problems. They are particularly useful for multi-dimensional problems.

Chapters 8 and 9 essentially repeat material in Chapters 5 and 6, but in polar, cylindrical, and spherical coordinates.

In Chapters 10 and 11, we introduce three further transforms for solving partial differential equations, Laplace, Fourier and Hankel. Chapter 10 contains a thorough presentation of the theory of Laplace transforms, particularly as it pertains to solving ordinary and partial differential equations. The transform is applied to PDEs on finite and infinite spatial domains. Fourier transforms, and Fourier sine and cosine transforms, in Chapter 11 are developed from Fourier integrals. They are then applied to problems on infinite and semi-infinite domains. Hankel transforms are applied to problems in polar and cylindrical coordinates.

Green's functions for ordinary differential equations and partial differential equations are discussed in Chapters 12 and 13. Chapter 13 utilizes separation techniques from Chapter 6, Section 9.1, and Chapter 12.

Chapters 14, 15, and 16 provide an introduction to numerical techniques for approximating solutions to PDEs, namely finite differences, weighted residuals, and finite elements.

To work through most sections of the book, students require a first course in ordinary differential equations and an introduction to advanced calculus. Sections 10.3–10.5, and Chapter 11 assume a working knowledge of the theory of complex functions.

There are six appendices of material. Appendices A and B give proofs of convergence theorems for Fourier series and Fourier integrals. Appendix C reviews ever so briefly those aspects of vector integral calculus that are used throughout the book. Appendix D contains discussions on parts of the theory of complex functions necessary in the chapters on Laplace and Fourier transforms. Appendix E contains numerical answers to all exercises. Appendix F is a reference to examples and exercises in Chapters 2-13 that contain physical applications of PDEs. Hopefully, it will help readers locate a problem in which they have a particular interest.

We believe that exercises are of the utmost importance to a student's learning. There must be straightforward problems to reinforce fundamentals and more difficult problems to challenge enterprising students. We have attempted to provide more than enough of each type. Problems in each set of exercises are graded from easy to difficult, and numerical answers to all exercises are provided in Appendix E. Exercise sets in sixteen sections (4.2, 4.3, 6.2, 6.3, 6.4, 7.2, 7.3, 9.1, 9.2, 10.2, 10.4, 10.5, 11.4, 11.6, 11.7, and 12.4) stress applications. They have been divided into four parts:

Part A — Heat Conduction
Part B — Vibrations
Part C — Potential, Steady-state Heat Conduction, Static Deflections of Membranes
Part D — General Results

Students interested in heat conduction could concentrate on problems from Part A. Students interested in mechanical vibrations will find problems in Part B particularly appropriate. All students can profit from problems in Part C, since every problem therein, although stated in terms of one of the three applications, is easily interpretable in terms of the other two. We recommend the exercises in Part D to all students.

A solutions manual containing solutions to all exercises is available.