Math 1010 Assignment 1 Solutions Winter 2014

1. (a)

$$\begin{aligned} 12\left(\frac{2x}{3}\right) + (12)3(x-1) &= (12)\frac{5(1-2x)}{4} + 12(2x) \\ 8x + 36(x-1) &= 15(1-2x) + 24x \\ 8x + 36x - 36 &= 15 - 30x + 24x \\ 44x + 6x &= 15 + 36 \\ 50x &= 51 \\ x &= \frac{51}{50} \end{aligned}$$

(b)

$$\begin{aligned} 4(x-1) + 3x &= 7x + 24 \\ 4x - 4 + 3x &= 7x + 24 \\ 7x - 7x &= 24 + 4 \\ 0 &= 28 \end{aligned}$$

The equation has no solutions.

(c)

$$\begin{aligned} 2(1-4y) + 3y + 2 &= -5y + 4 \\ 2 - 8y + 3y + 2 &= -5y + 4 \\ -5y + 5y &= 4 - 4 \\ 0 &= 0 \end{aligned}$$

All real numbers satisfy the equation.

2. (a)

$$\begin{aligned} 6\left(\frac{5x}{2}\right) + (6)2(3+x) &\geq (6)\frac{4(x-5)}{3} \\ 15x + 12(3+x) &\geq 8(x-5) \\ 15x + 36 + 12x &\geq 8x - 40 \\ 27x - 8x &\geq -40 - 36 \\ 19x &\geq -76 \\ x &\geq -\frac{76}{19} \\ x &\geq -4 \end{aligned}$$

(b)

$$\begin{aligned} 3(x-5) - x + 10 &\geq 2x + 6 \\ 3x - 15 - x + 10 &\geq 2x + 6 \\ 2x - 2x &\geq 6 + 5 \\ 0 &\geq 11 \end{aligned}$$

The inequality has no solutions.

(c)

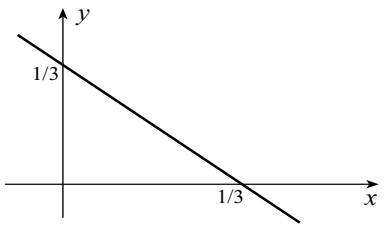
$$\begin{aligned} 10y + 3(4 - 2y) &< 2y + 2(y + 15) \\ 10y + 12 - 6y &< 2y + 2y + 30 \\ 4y - 4y &< 30 - 12 \\ 0 &< 18 \end{aligned}$$

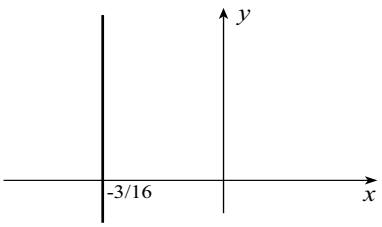
All real numbers satisfy the inequality.

3. (a) Because of the term $2y^2$, this equation does not represent a straight line.

(b) Because the square root is above the x, this equation does not represent a straight line.

(c) This is a straight line. It is shown in the left figure below.





(d) This is the equation of a straight line. To draw it, we rewrite the equation in the form

$$3(1 - 5x) = 3(2) + 3\left(\frac{x}{3}\right) \quad \Longrightarrow \quad 3 - 15x = 6 + x \quad \Longrightarrow \quad -16x = 3 \quad \Longrightarrow \quad x = -\frac{3}{16}.$$

It is shown in the right figure above.

4. When we write the equation of the given line in the form $y = \frac{5}{6}x + \frac{777}{6}$, we see that its slope is the $5/6$. The slope of the required line is therefore $-6/5$. Its equation is

$$\begin{aligned} y + 3 &= -\frac{6}{5}(x - 2) \\ 5(y + 3) &= -6(x - 2) \\ 5y + 15 - 6x + 12 & \\ 6x + 5y &= -3 \end{aligned}$$

5. When we write the equation of the given line in the form $y = -\frac{3}{2}x - 36$, we see that its slope is the $-3/2$. The slope of the required line is therefore $-3/2$ also. To find the point of interesection of the lines $6x - 4y = -11$ and $x = 2y$, we substitute $x = 2y$ in the first equation

$$6(2y) - 4y = -11 \quad \Longrightarrow \quad 12y - 4y = -11 \quad \Longrightarrow \quad 8y = -11 \quad \Longrightarrow \quad y = -\frac{11}{8}.$$

This gives $x = -11/4$. The point of intersection is therefore $(-11/4, -11/8)$. The equation of the required line is therefore

$$\begin{aligned} y + \frac{11}{8} &= -\frac{3}{2}\left(x + \frac{11}{4}\right) \\ 8(y) + 8\left(\frac{11}{8}\right) &= 8\left[-\frac{3}{2}\left(x + \frac{11}{4}\right)\right] \\ 8y + 11 &= -12\left(x + \frac{11}{4}\right) \end{aligned}$$

$$8y + 11 = -12x - 33$$
$$12x + 8y = -44$$
$$3x + 2y = -11$$

6. The feasible set is shown below.