

MATH 1010 Assignment 3 Summer 2014

Solutions to Assignment 3

1. (a) This matrix is in row echelon form.

(b) This matrix is not in row echelon form.

$$\left( \begin{array}{ccc|c} 2 & 1 & 3 & 4 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 1 & 4 \end{array} \right) \begin{array}{l} \\ R_3 \rightarrow -2R_2 + R_3 \end{array} \rightarrow \left( \begin{array}{ccc|c} 2 & 1 & 3 & 4 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & -7 & -2 \end{array} \right) \begin{array}{l} R_1 \rightarrow R_1/2 \\ R_3 \rightarrow R_3/(-7) \end{array} \rightarrow \left( \begin{array}{ccc|c} 1 & 1/2 & 3/2 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 1 & 2/7 \end{array} \right)$$

(c) This matrix is not in row echelon form.

$$\left( \begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & 0 \\ 0 & 8 & 6 \end{array} \right) \begin{array}{l} \\ R_2 \rightarrow R_2/4 \\ R_4 \rightarrow -2R_2 + R_4 \end{array} \rightarrow \left( \begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3/4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)$$

2. (a) This matrix is not in reduced row echelon form.

$$\left( \begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 0 & 5 & 1 \\ 0 & 1 & 2 & 2 \end{array} \right) \begin{array}{l} \\ R_2 \rightarrow R_3 \\ R_3 \rightarrow R_2 \end{array} \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 5 & 1 \end{array} \right) \begin{array}{l} R_1 \rightarrow -2R_2 + R_1 \\ R_3 \rightarrow R_3/5 \end{array} \\ \rightarrow \left( \begin{array}{ccc|c} 1 & 0 & -7 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1/5 \end{array} \right) \begin{array}{l} R_1 \rightarrow 7R_3 + R_1 \\ R_2 \rightarrow -2R_3 + R_2 \end{array} \rightarrow \left( \begin{array}{ccc|c} 1 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 8/5 \\ 0 & 0 & 1 & 1/5 \end{array} \right)$$

(b) This matrix is in reduced row echelon form.

(c) This matrix is not in reduced row echelon form.

$$\left( \begin{array}{ccc|c} 3 & 0 & 1 & -6 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 4 & 4 \end{array} \right) \begin{array}{l} \\ R_3 \rightarrow R_3/4 \end{array} \rightarrow \left( \begin{array}{ccc|c} 3 & 0 & 1 & -6 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} R_1 \rightarrow -R_3 + R_1 \\ R_2 \rightarrow -2R_3 + R_2 \end{array} \\ \rightarrow \left( \begin{array}{ccc|c} 3 & 0 & 0 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} R_1 \rightarrow R_1/3 \\ \end{array} \rightarrow \left( \begin{array}{ccc|c} 1 & 0 & 0 & -7/3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

3. (a)  $\left( \begin{array}{ccc|c} 2 & 1 & -1 & 6 \\ 3 & -2 & 4 & 7 \\ 7 & 0 & 2 & 1 \end{array} \right) \begin{array}{l} \\ R_2 \rightarrow -R_1 + R_2 \end{array} \rightarrow \left( \begin{array}{ccc|c} 2 & 1 & -1 & 6 \\ 1 & -3 & 5 & 1 \\ 7 & 0 & 2 & 1 \end{array} \right) \begin{array}{l} R_1 \rightarrow R_2 \\ R_2 \rightarrow R_1 \end{array}$

$$\rightarrow \left( \begin{array}{ccc|c} 1 & -3 & 5 & 1 \\ 2 & 1 & -1 & 6 \\ 7 & 0 & 2 & 1 \end{array} \right) \begin{array}{l} \\ R_2 \rightarrow -2R_1 + R_2 \\ R_3 \rightarrow -7R_1 + R_3 \end{array} \rightarrow \left( \begin{array}{ccc|c} 1 & -3 & 5 & 1 \\ 0 & 7 & -11 & 4 \\ 0 & 21 & -33 & -6 \end{array} \right) \begin{array}{l} \\ R_3 \rightarrow -3R_2 + R_3 \end{array}$$

$$\rightarrow \left( \begin{array}{ccc|c} 1 & -3 & 5 & 1 \\ 0 & 7 & -11 & 4 \\ 0 & 0 & 0 & -18 \end{array} \right)$$

When we convert to equations, the last line gives  $0 = -18$ . Hence, the equations do not have a solution.



$$\begin{aligned}
& \text{(b)} \left( \begin{array}{cccc|c} 1 & 1 & -2 & 2 & 1 \\ 3 & 0 & -1 & 5 & 2 \\ 4 & 1 & -2 & 2 & 2 \\ -2 & -2 & 3 & 1 & -1 \end{array} \right) \begin{array}{l} R_2 \rightarrow -3R_1 + R_2 \\ R_3 \rightarrow -4R_1 + R_3 \\ R_4 \rightarrow 2R_1 + R_4 \end{array} \rightarrow \left( \begin{array}{cccc|c} 1 & 1 & -2 & 2 & 1 \\ 0 & -3 & 5 & -1 & -1 \\ 0 & -3 & 6 & -6 & -2 \\ 0 & 0 & -1 & 5 & 1 \end{array} \right) R_3 \rightarrow -R_2 + R_3 \\
& \rightarrow \left( \begin{array}{cccc|c} 1 & 1 & -2 & 2 & 1 \\ 0 & -3 & 5 & -1 & -1 \\ 0 & 0 & 1 & -5 & -1 \\ 0 & 0 & -1 & 5 & 1 \end{array} \right) \begin{array}{l} R_1 \rightarrow 2R_3 + R_1 \\ R_2 \rightarrow -5R_3 + R_2 \\ R_4 \rightarrow R_3 + R_4 \end{array} \rightarrow \left( \begin{array}{cccc|c} 1 & 1 & 0 & -8 & -1 \\ 0 & -3 & 0 & 24 & 4 \\ 0 & 0 & 1 & -5 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) R_2 \rightarrow R_2/(-3) \\
& \rightarrow \left( \begin{array}{cccc|c} 1 & 1 & 0 & -8 & -1 \\ 0 & 1 & 0 & -8 & -4/3 \\ 0 & 0 & 1 & -5 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) R_1 \rightarrow -R_2 + R_1 \rightarrow \left( \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & -8 & -4/3 \\ 0 & 0 & 1 & -5 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)
\end{aligned}$$

When we convert to equations,

$$x = \frac{1}{3}, \quad y - 8w = -\frac{4}{3}, \quad z - 5w = -1.$$

The solution is  $x = \frac{1}{3}$ ,  $y = -\frac{4}{3} + 8w$ ,  $z = -1 + 5w$ , where  $w$  is arbitrary.

$$\begin{aligned}
& \text{5. (a)} \left( \begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ -1 & 5 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} R_1 \rightarrow R_2 \\ R_2 \rightarrow R_1 \end{array} \rightarrow \left( \begin{array}{ccc|ccc} 1 & -1 & 2 & 0 & 1 & 0 \\ 2 & 1 & 3 & 1 & 0 & 0 \\ -1 & 5 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} R_2 \rightarrow -2R_1 + R_2 \\ R_3 \rightarrow R_1 + R_3 \end{array} \\
& \rightarrow \left( \begin{array}{ccc|ccc} 1 & -1 & 2 & 0 & 1 & 0 \\ 0 & 3 & -1 & 1 & -2 & 0 \\ 0 & 4 & 2 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} R_3 \rightarrow -R_2 + R_3 \\ R_3 \rightarrow R_2 \end{array} \\
& \rightarrow \left( \begin{array}{ccc|ccc} 1 & -1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 3 & -1 & 3 & 1 \\ 0 & 3 & -1 & 1 & -2 & 0 \end{array} \right) \begin{array}{l} R_1 \rightarrow R_2 + R_1 \\ R_3 \rightarrow -3R_2 + R_3 \end{array} \rightarrow \left( \begin{array}{ccc|ccc} 1 & 0 & 5 & -1 & 4 & 1 \\ 0 & 1 & 3 & -1 & 3 & 1 \\ 0 & 0 & -10 & 4 & -11 & -3 \end{array} \right) R_3 \rightarrow R_3/(-10) \\
& \rightarrow \left( \begin{array}{ccc|ccc} 1 & 0 & 5 & -1 & 4 & 1 \\ 0 & 1 & 3 & -1 & 3 & 1 \\ 0 & 0 & 1 & -2/5 & 11/10 & 3/10 \end{array} \right) \begin{array}{l} R_1 \rightarrow -5R_3 + R_1 \\ R_2 \rightarrow -3R_3 + R_2 \end{array} \rightarrow \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3/2 & -1/2 \\ 0 & 1 & 0 & 1/5 & -3/10 & 1/10 \\ 0 & 0 & 1 & -2/5 & 11/10 & 3/10 \end{array} \right)
\end{aligned}$$

The inverse matrix is  $A^{-1} = \begin{pmatrix} 1 & -3/2 & -1/2 \\ 1/5 & -3/10 & 1/10 \\ -2/5 & 11/10 & 3/10 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 10 & -15 & -5 \\ 2 & -3 & 1 \\ -4 & 11 & 3 \end{pmatrix}$ .

$$\begin{aligned}
& \text{(b)} \left( \begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 4 & 5 & 2 & 0 & 1 & 0 \\ -2 & 1 & 4 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} R_1 \rightarrow R_3 + R_1 \\ R_2 \rightarrow -4R_1 + R_2 \\ R_3 \rightarrow 2R_1 + R_3 \end{array} \\
& \rightarrow \left( \begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 1 \\ 0 & -7 & -10 & -4 & 1 & -4 \\ 0 & 7 & 10 & 2 & 0 & 3 \end{array} \right) \begin{array}{l} R_3 \rightarrow R_2 + R_3 \end{array}
\end{aligned}$$

This shows that the matrix does not have an inverse.