Solutions to MATH1010 Assignment 4

1.

	Path	Circuit	Euler Path	Euler Circuit	Hamiltonian Circuit
(a)	x	x			
(b)	x	x			x
(c)					
(d)	x	x	x	x	
(e)					

2. (a) Because the graph has exactly two nodes with odd degrees, the graph has Euler paths, but no Euler circuits.

(b) Because the graph has three nodes with odd degrees, the graph has no Euler paths nor Euler circuits.

(c) Because all nodes have even degree, the graph has Euler circuits.

3. The graph to the right represents the two shores, the four islands, and the seven bridges. Since two nodes have odd degree, there are no Euler circuits for the graph, but there are Euler paths. This means that it is not possible to cross all seven bridges and return to the same starting point, but it is possible to start from one point, cross all bridges, and return to a different point.

4. The root diagram below shows that there are eight Hamiltonian circuits starting at node 1. Since the last four are the first four in reverse, we calculate distances for the first four. The Hamiltonian circuits that minimize distance are (1,2,3,4,5,1) and (1,5,4,3,2,1).

Circuit	Length
(1,2,3,4,5,1)	100+200+250+150+200=900
(1,2,5,4,3,1)	100+200+150+250+500=1200
(1,3,2,5,4,1)	500+200+200+150+400=1450
(1,4,3,2,5,1)	400+250+200+200+200=1250
(1,5,4,3,2,1)	
(1,3,4,5,2,1)	
(1,4,5,2,3,1)	
(1,5,2,3,4,1)	

5. The adjacency matrix is $\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$

6. The graph is

7. (a) $A^2 = \begin{pmatrix} 4 & 1 & 2 & 3 & 2 \\ 1 & 2 & 2 & 1 & 2 \\ 2 & 2 & 3 & 2 & 2 \\ 3 & 1 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

(b) This is the $(2, 4)$ entry of A^2. It is 1.

(c) This is the $(3, 3)$ entry of A^2. It is 3.

(d) This is the $(4, 1)$ entry of A^3. It is $(1, 1, 1, 0, 1) \begin{pmatrix} 4 \\ 1 \\ 2 \\ 3 \\ 2 \end{pmatrix} = 9$.

(e) This is the $(3, 2)$ entry of $A + A^2$. It is $0 + 2 = 2$.

(f) This is the $(1, 5)$ entry of $A + A^2 + A^3$. It is $1 + 2 + (0, 1, 1, 1, 1) \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 3 \end{pmatrix} = 3 + 9 = 12$.