MATH 1210 Assignment 2 Fall 2018

1. Find all exponential representations for

(a) $(-\sqrt{3}-i)^6$ (b) $\dfrac{(1+i)^{14}(2+2\sqrt{3}i)^4}{4^6 i(1-i)}$

2. What is the remainder when $P(x) = (1-2i)x^3 + 3ix^2 + 4x - 2i$ is divided by $2x - 1 + 3i$?

3. Find h and k so that remainders are $1291/2$ and $123/16$ when $x^4 + hx^2 - x + k$ is divided by $x+5$ and $2x-3$, respectively.

4. (a) Show that $x = -1 + 2i$ is a zero of the polynomial

$$P(x) = x^4 + 2x^3 + (5+i)x^2 + 2ix + 5i.$$

(b) Factor $P(x)$ into linear factors.

5. In each part of this question: (i) use Descartes' rules of signs to state the number of possible positive and negative zeros of the polynomial; (ii) use the bounds theorem to find bounds for zeros of the polynomial; (iii) use the rational root theorem to list all possible rational zeros of the polynomial. Take the results of (i) and (ii) into account in (iii).

(a) $15x^8 - 2x^4 + 3x - 12$ (b) $24x^4 - 13x^3 + 2x^2 - 5x + 21$

6. In each part of this question, use the procedure of Problem 5 to find all roots of the equation:

(a) $12x^4 + 7x^3 + 2x^2 + 7x - 10 = 0$

(b) $x^4 + 2x^3 - 41x^2 - 42x + 360 = 0$

(c) $2x^6 - x^5 + 4x - 2 = 0$

(d) $x^6 + x^3 + 1 = 0$

7. Prove that if a_n is greater than $2|a_{n-1}|$, $2|a_{n-2}|$, ..., $2|a_0|$, then every zero of the polynomial $P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ must satisfy

$$|x| < \frac{3}{2}.$$

8. Prove that if $P(x)$ is a polynomial having only even powers of x, and $P(a) = 0$, then $P(x)$ is divisible by $x^2 - a^2$.