MATH 1210 (FALL TERM 2019)
ASSIGNMENT THREE

Please complete the entire assignment, staple it to the Honesty Declaration Form (which should serve as the front page) and submit it to your instructor on Friday, November 29th, 2019 at the beginning of the class. Only some of the questions will be marked. Late assignments will not be accepted. You must **show your work** in reasonable detail in order to get marks.

Q1. Consider the matrix

$$A = \begin{bmatrix} 3 & -1 & 4 & 7 \\ 1 & 2 & -5 & 8 \end{bmatrix}.$$

- Find the reduced row-echelon form (RREF) of A as follows: First divide the first row by 3, i.e., perform the operation $R_1 \leftarrow R_1/3$. Now use the 1 in the $(1,1)$ position to make the $(2,1)$ entry zero and so on. Show all of your row operations carefully.
- Now, starting from A again, find the reduced row-echelon form (RREF) of A as follows: Switch the two rows, i.e., perform the operation $R_1 \leftrightarrow R_2$. Then use the 1 in the $(1,1)$ position to make the $(2,1)$ entry zero and so on. Show all of your row operations carefully.

By either method, the final RREF should come out to be the *same*. If this does not work out, then you have made a mistake somewhere.

Q2. Consider the three vectors

$$\mathbf{u} = \langle a+3, 1, 1\rangle, \quad \mathbf{v} = \langle 1, a+2, 2\rangle, \quad \mathbf{w} = \langle 1, 2, a+2\rangle,$$

where a is a real number.

(1) Find all values of a such that the vectors are linearly dependent.

(2) For each of the values of a found in (1), express $\mathbf{v}$ as a linear combination of the vectors $\mathbf{u}$ and $\mathbf{w}$.

Q3. It is given to you that the matrix

$$A = \begin{bmatrix} a+b-3 & b+3a & 0 & 1 \\ 0 & 0 & 1 & 4a-7b \\ 0 & 0 & 3a+2b-5 & 0 \end{bmatrix}$$

is in reduced row-echelon form (RREF). Find the values of a and b.

Q4. Consider the following system of equations:

$$x + y + z = 1, \qquad -2x + y - 3z = 0, \qquad 5x - 8y + 11z = 3.$$

(1) Solve this system using Gauss-Jordan elimination. That is to say, convert the augmented matrix of the system to RREF and then read off the solutions.
(2) Now solve the same system using Cramer's rule.

Of course, your solutions in (1) and (2) must agree with each other. If they do not, then you have made a mistake somewhere.

Q5. Consider the matrix

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Find $\det(M)$.
Hint: This is very messy to do if you try a straightforward cofactor expansion. Instead, use row-operations to convert it into a form which can be calculated more easily. The question is meant to be tricky.