Math 1210 Tutorial 11

1. Use properties 9.2 to prove or disprove that the following transformations are linear:

(a) $T(\mathbf{v}) = k\mathbf{v}$, where k is a nonzero constant

(b) $T(\mathbf{v}) = \mathbf{v} + \mathbf{u}$, where $\mathbf{u}$ is a constant, nonzero vector.

2. If $\mathbf{v}' = T(\mathbf{v})$ is a linear transformation from R^3 to R^3 where

$$\begin{aligned} v_1' &= 3v_1 - 2v_2, \\ v_2' &= 4v_1 + 3v_2 + v_3, \\ v_3' &= -v_1 + 2v_2 + 3v_3, \end{aligned}$$

(a) find $T\langle 2, -1, 3\rangle$,

(b) find $\mathbf{v}$ if $\mathbf{v}' = \langle 1, 1, -1\rangle$,

(c) find all vectors so that $T(\mathbf{v}) = 2\mathbf{v}$.

3. You are told that the characteristic equation for a matrix is

$$6\lambda^4 + 11\lambda^3 - 4\lambda^2 + 11\lambda - 10 = 0.$$

What are the eigenvalues of the matrix?

4. What are eigenvalues and eigenvectors for an identity matrix?

5. Find all eigenvalues and all corresponding eigenvectors for each of the following matrices:

(a) $\begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$

6. Prove that if zero is an eigenvalue for a matrix, then the matrix cannot have an inverse.

Answers:

1. (a) Linear (b) Not linear

2. (a) $\langle 8, 8, 5\rangle$ (b) $\langle 15/47, -1/47, -10/47\rangle$ (c) $\langle 0, 0, 0\rangle$

3. $-2/5$, $2/3$, $\pm i$

4. The only eigenvalue is 1, and every vector is an eigenvector.

5. (a) $\lambda = 1$, $\mathbf{v} = v_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + v_3 \begin{pmatrix} -1/2 \\ 0 \\ 1 \end{pmatrix}$;

$\lambda = 10$, $\mathbf{v} = v_3 \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

(b) $\lambda = 1, \quad \mathbf{v} = v_3 \begin{pmatrix} -1/2 \\ 1/2 \\ 1 \end{pmatrix};$

$\lambda = 2, \quad \mathbf{v} = v_3 \begin{pmatrix} -1/2 \\ 1/4 \\ 1 \end{pmatrix};$

$\lambda = 3, \quad \mathbf{v} = v_3 \begin{pmatrix} -1/4 \\ 1/4 \\ 1 \end{pmatrix}$