MATH 1210 Tutorial 3

1. Express each of the following in exponential form. Your final answer should have the principal value of the argument.

(a) $(3+3\sqrt{3}i)^7 e^{5\pi i/6}$ (b) $\dfrac{(1+i)e^{3\pi i/4}}{3e^{-\pi i/3}}$

2. Use Euler's identity and DeMoivre's theorem to prove the well-known double angle formula

$$\cos 2\theta = \cos^2\theta - \sin^2\theta.$$

3. Use Euler's identity to prove that

$$\cos\theta = \frac{e^{\theta i}+e^{-\theta i}}{2}, \qquad \sin\theta = \frac{e^{\theta i}-e^{-\theta i}}{2i}.$$

4. Find exact values for all solutions of the following equations. Express final answers in Cartesian form.

(a) $2x^4+3x^2-1=0$ (b) $z^4=-4i$

5. Find the fifth roots of $-2-2i$. Write answers in Cartesian form.

Answers:

1.(a) $6^7 e^{-5\pi i/6}$ (b) $(\sqrt{2}/3)e^{-2\pi i/3}$

4.(a) $\pm\dfrac{\sqrt{\sqrt{17}-3}}{2}$, $\pm\dfrac{\sqrt{\sqrt{17}+3}}{2}\,i$

(b) $\sqrt{2}\cos\left(\frac{\pi}{8}\right)-\sqrt{2}\sin\left(\frac{\pi}{8}\right)i$, $\sqrt{2}\cos\left(\frac{3\pi}{8}\right)+\sqrt{2}\sin\left(\frac{3\pi}{8}\right)i$,
$\sqrt{2}\cos\left(\frac{7\pi}{8}\right)+\sqrt{2}\sin\left(\frac{7\pi}{8}\right)i$, $\sqrt{2}\cos\left(\frac{11\pi}{8}\right)+\sqrt{2}\sin\left(\frac{11\pi}{8}\right)i$

5. $2^{3/10}\cos\left(\frac{3\pi}{20}\right)-2^{3/10}\sin\left(\frac{3\pi}{20}\right)i$, $2^{-1/5}+2^{-1/5}\,i$, $2^{3/10}\cos\left(\frac{13\pi}{20}\right)+2^{3/10}\sin\left(\frac{13\pi}{20}\right)i$,
$2^{3/10}\cos\left(\frac{21\pi}{20}\right)+2^{3/10}\sin\left(\frac{21\pi}{20}\right)i$, $2^{3/10}\cos\left(\frac{29\pi}{20}\right)+2^{3/10}\sin\left(\frac{29\pi}{20}\right)i$