

## Math 1210 Tutorial 5

1. Find the following matrix products, if they exist:

$$(a) \begin{pmatrix} 2 & -3 & 4 \\ 0 & 1 & -2 \\ 3 & -4 & 3 \end{pmatrix} \begin{pmatrix} 3 & 4 & -2 \\ 6 & 2 & 0 \\ 5 & -1 & 3 \end{pmatrix} \quad (b) (1, -2, 3)(3, 4, 5)$$

$$(c) (1, -2, 3) \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \quad (d) \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} (3, 4, 5) \quad (e) \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

2. If

$$A = \begin{pmatrix} 2 & 1 \\ -2 & 3 \\ 5 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 5 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & 5 \\ 2 & 6 \\ 0 & 7 \end{pmatrix},$$

find the matrix  $X$  such that

$$3(X + I) - 2AC^T = B^2.$$

3. Prove or disprove the following equation for square matrices  $A$  and  $B$  of the same size

$$(A - B)(A + B) = A^2 - B^2.$$

4. Find the matrix  $A = (a_{ij})_{3 \times 3}$  if  $a_{ij} = i^2 - j$ .

5. If  $\mathbf{u} = \langle 3, -2, 4 \rangle$  and  $\mathbf{v} = \langle -3, 6, 2 \rangle$ , find:

$$(a) 2\mathbf{u} - 4\mathbf{v} \quad (b) |\mathbf{u}|\hat{\mathbf{v}} + 3(\mathbf{u} \cdot \mathbf{v})\mathbf{u} \quad (c) 2\mathbf{u} \times (-3\mathbf{v})$$

6. Prove that  $\mathbf{v} \times \mathbf{u} = -\mathbf{u} \times \mathbf{v}$ .

**Answers:**

$$1. (a) \begin{pmatrix} 8 & -2 & 8 \\ -4 & 4 & -6 \\ 0 & 1 & 3 \end{pmatrix} \quad (b) \text{Not possible} \quad (c) (10)$$

$$(d) \begin{pmatrix} 3 & 4 & 5 \\ -6 & -8 & -10 \\ 9 & 12 & 15 \end{pmatrix} \quad (e) \text{Not possible}$$

$$2. \begin{pmatrix} 16/3 & 29/3 & 7 \\ 26/3 & 26/3 & 58/3 \\ 80/3 & 64/3 & 34/3 \end{pmatrix} \quad 3. \text{Not true} \quad 4. \begin{pmatrix} 0 & -1 & -2 \\ 3 & 2 & 1 \\ 8 & 7 & 6 \end{pmatrix}$$

$$5. (a) \langle 18, -28, 0 \rangle \quad (b) \left\langle \frac{-3\sqrt{29}}{7} - 117, \frac{6\sqrt{29}}{7} + 78, \frac{2\sqrt{29}}{7} - 156 \right\rangle \quad (c) \langle 168, 108, -72 \rangle$$