

Math 1210 Tutorial 8

Use Gauss-Jordan elimination to find all solutions for each of the following systems of equations:

1.

$$\begin{aligned}2x + 3y - 4z + w &= 16, \\ y + 2z - 3w &= -12, \\ 3x - y + 2w &= 9, \\ 2x + y + z &= 3.\end{aligned}$$

2.

$$\begin{aligned}2x + 3y - 4z + w &= 3, \\ x - 2y + z &= 6, \\ 3x + y + w &= 4, \\ 6x + 2y - 3z + 2w &= 13.\end{aligned}$$

3. Find basic solutions, which consist of integers only, for the following system of homogeneous equations

$$\begin{aligned}x + 5y + 3z - 5w &= 0, \\ 2x - y + 3z - 4w &= 0.\end{aligned}$$

Are they unique?

4. (a) Show that solutions of the system of nonhomogeneous equations

$$\begin{aligned}2x - y + 3z + 5w &= 3, \\ x + 3y - 2z + w &= -2, \\ 3x + 2y + z + 6w &= 1,\end{aligned}$$

can be expressed in the form

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = z \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \frac{w}{7} \begin{pmatrix} -16 \\ 3 \\ 0 \\ 7 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}.$$

(b) Are $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -16 \\ 3 \\ 0 \\ 7 \end{pmatrix}$ basic solutions of the system? Are they even solutions of the system?

(c) Is $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$ a solution of the system? Is it a basic solution?

Answers:

1. $x = 1, y = 2, z = -1, w = 4$

2. $x = \frac{47}{21} - \frac{2t}{7}, y = -\frac{19}{7} - \frac{t}{7}, z = -\frac{5}{3}, w = t$

3. $\begin{pmatrix} -18 \\ -3 \\ 11 \\ 0 \end{pmatrix}, \begin{pmatrix} 25 \\ 6 \\ 0 \\ 11 \end{pmatrix}$ No

4. (a) No, No (b) Yes, No