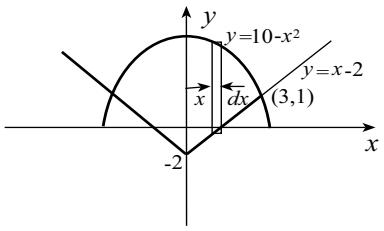
MATH1710 Test1 **February 15, 2017** **50 minutes**

Values

9 **1.** Find the area bounded by the curves

$$y = 10 - x^2, \quad y = |x| - 2.$$

$$\begin{aligned} A &= 2\int_0^3 [(10 - x^2) - (x - 2)]\, dx \\ &= 2\int_0^3 (12 - x - x^2)\, dx \\ &= 2\left\{12x - \frac{x^2}{2} - \frac{x^3}{3}\right\}_0^3 \\ &= 45 \end{aligned}$$

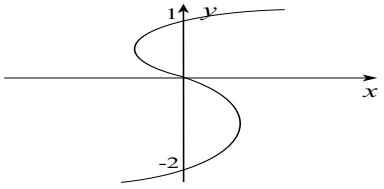


3 **2.** In one or two sentences, explain the difference between **an** antiderivative of a function $f(x)$, and **the** indefinite integral of the function.

An antiderivative of $f(x)$ is any function whose first derivative is $f(x)$. The indefinite integral of $f(x)$ is all functions whose first derivatives are $f(x)$.

6 **3.** Set up, but do **NOT** evaluate, a definite integral for the length of the curve $x = y^3 - 2y^2$ between the points $(-16, -2)$ and $(-1, 1)$.

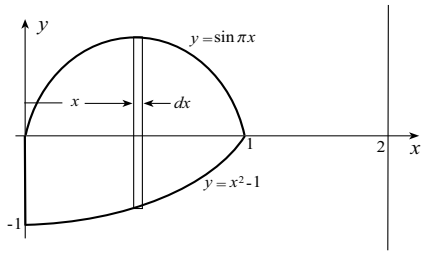
$$\begin{aligned} L &= \int_{-2}^{1} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \\ &= \int_{-2}^{1} \sqrt{1 + (3y^2 - 4y)^2}\, dy \end{aligned}$$



9 **4.** Set up, but do **NOT** evaluate, a definite integral for the volume of the solid of revolution when the area bounded by the curves

$$y = \sin \pi x, \qquad x = \sqrt{1+y}, \qquad x = 0$$

is rotated about the line $x = 2$.



$$V = \int_0^1 2\pi(2-x)[\sin \pi x - (x^2-1)]\,dx$$
$$= \int_0^1 (2-x)(\sin \pi x - x^2 + 1)\,dx$$

13 **5.** The ends of a horizontal water trough with length 5 metres are parabolic with width 3/2 metres, and depth 1 metre at the centre (see figure below). If the depth of water in the trough is 1/2 metre, set up, but do **NOT** evaluate, a definite integral to find the work required to empty the trough to a height 1/2 metre above the top of the trough. Replace all physical constants with their numerical values.

$$V = \int_0^{1/2} (3/2-y)\rho g(5)(2x)\,dy$$
$$= 1000(10)(9.81)\int_0^{1/2} (3/2-y)\left(\frac{3\sqrt{y}}{4}\right)dy \text{ J}$$