

MATH 1710 Tutorial 8

In problems 1–6, evaluate the integral.

$$1. \int \cot^5(2x) \csc^3(2x) dx \qquad 2. \int \sqrt{\tan x} \sec^4 x dx$$

$$3. \int \frac{\tan^3 x}{\sec^4 x} dx \qquad 4. \int \sin^2 x \cos 2x dx$$

$$5. \int \sin 3x \cos 7x dx \qquad 6. \int_0^{\pi/4} \sin^4 x dx$$

7. Evaluate the integral $\int \frac{x^3}{\sqrt{9-4x^2}} dx$ with:

- (a) integration by parts,
- (b) the substitution $u = 9 - 4x^2$,
- (c) the substitution $u = \sqrt{9 - 4x^2}$,
- (d) a trigonometric substitution.

Verify that all answers are the same.

In problems 8–11, evaluate indefinite integral.

$$8. \int \frac{1}{x\sqrt{4-9x^2}} dx \qquad 9. \int \frac{\sqrt{9-2x^2}}{x^4} dx$$

$$10. \int \frac{\sqrt{x^2-4}}{x^3} dx \qquad 11. \int \frac{x^5}{\sqrt{x^2+9}} dx$$

Answers

$$1. -\frac{1}{14} \csc^7(2x) + \frac{1}{5} \csc^5(2x) - \frac{1}{6} \csc^3(2x) + C \qquad 2. \frac{2}{3} \tan^{3/2} x + \frac{2}{7} \tan^{7/2} x + C$$

$$3. \frac{1}{4} \sin^4 x + C \qquad 4. \frac{1}{4} \sin 2x - \frac{x}{4} - \frac{1}{16} \sin 4x + C$$

$$5. -\frac{1}{20} \cos 10x + \frac{1}{8} \cos 4x + C \qquad 6. \frac{3\pi - 8}{32}$$

$$7.(a) -\frac{x^2}{4} \sqrt{9-4x^2} - \frac{1}{24} (9-4x^2)^{3/2} + C \quad (b),(c),(d) \frac{1}{48} (9-4x^2)^{3/2} - \frac{9}{16} \sqrt{9-4x^2} + C$$

$$8. \frac{1}{2} \ln \left| \frac{2 - \sqrt{4-9x^2}}{x} \right| + C \qquad 9. -\frac{(9-2x^2)^{3/2}}{27x^3} + C$$

$$10. \frac{1}{4} \operatorname{Sec}^{-1} \left(\frac{x}{2} \right) - \frac{\sqrt{x^2-4}}{2x^2} + C \qquad 11. \frac{\sqrt{x^2+9}}{5} (216 - 12x^2 + x^4)$$