

MATH 1710 Tutorial 6

1. Find all solutions of the following equations:

(a) $3 \cos(2x + 1) = 1$ (b) $2 \tan(1 - 3x) = 3$ (c) $\sin^2 x - 3 \sin x + 1 = 0$

2.(a) Express the function $f(x) = 4 \cos 3x - 2 \sin 3x$, $x \geq 0$, in the form $A \sin(3x + \phi)$, where $A > 0$ and $0 < \phi < \pi$.

(b) Use the result in part (a) to find the third smallest value of x for which $f(x) = 0$.

3. Find first derivatives of the following functions. Simplify answers as much as possible.

(a) $f(x) = \sec^{-1} \sqrt{x^2 + 3x + 2}$

(b) $f(x) = \frac{1}{x} \csc^{-1}(3x) - \frac{\sqrt{9x^2 - 1}}{x}$

(c) $f(x) = \tan^{-1} \left(\frac{\sqrt{2}x}{\sqrt{1 + x^4}} \right)$

4. Find the largest value of the function

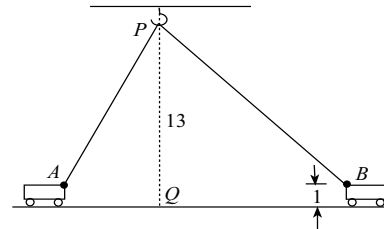
$$f(x) = \cot^{-1} \left(\frac{x}{a} \right) - \cot^{-1} \left(\frac{x}{b} \right)$$

on the interval $0 \leq x < \infty$, where $a > b > 0$ are constants.

5. A pulley P fixed to the ceiling (diagram below) is 13 metres above point Q on the floor directly below P . The ends of a rope with length 39 metres is attached to carts A and B , 1 metre above the floor. The rope is taut, and cart A moves away from Q at 2 metres per second.

(a) Find how fast cart B is moving when the horizontal distance between A and Q is 5 metres.

(b) Use the result in part (a) to find the rate of change of angle APB when the horizontal distance between A and Q is 5 metres.



Answers

1. (a) $\frac{1}{2} \left[-1 \pm \cos^{-1} \left(\frac{1}{3} \right) \right] + n\pi$ (b) $\frac{1}{3} \left[1 - \tan^{-1} \left(\frac{3}{2} \right) \right] + \frac{n\pi}{3}$

(c) $\sin^{-1} \left(\frac{3 - \sqrt{5}}{2} \right) + 2n\pi, \pi - \sin^{-1} \left(\frac{3 - \sqrt{5}}{2} \right) + 2n\pi$

2.(a) $2\sqrt{5} \sin[3x + \pi - \sin^{-1}(2/\sqrt{5})]$ (b) $\frac{1}{3} \left[2\pi + \sin^{-1}(2/\sqrt{5}) \right]$

3. (a) $\frac{2x + 3}{2(x^2 + 3x + 2)^{3/2} \sqrt{(x^2 + 3x + 2)^2 - 1}}$ (b) $-\frac{1}{x^2} \csc^{-1}(3x) - \frac{2}{x^2 \sqrt{9x^2 - 1}}$

(c) $\frac{\sqrt{2}(1 - x^2)}{(1 + x^2)\sqrt{1 + x^4}}$

4. $\cot^{-1} \sqrt{\frac{b}{a}} - \cot^{-1} \sqrt{\frac{a}{b}}$ 5.(a) -0.867 (b) 0.127 rad/sec