Math 1710 Test 1 **60 minutes**

Student Name - **Student Number -**

Circle your instructor's name T. Berry D. Trim

Values

9 **1.** Find the area bounded by the curves

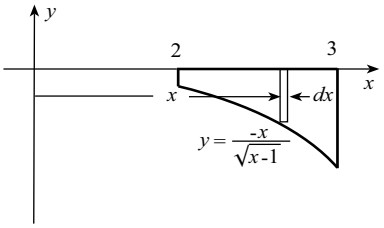
$$y = -\frac{x}{\sqrt{x-1}}, \quad y = 0, \quad x = 2, \quad x = 3.$$

$$A = \int_2^3 \frac{x}{\sqrt{x-1}}dx$$

If we set $u = x - 1$ and $du = dx$, then

$$A = \int_1^2 \frac{u+1}{\sqrt{u}}du = \left\{\frac{2}{3}u^{3/2} + 2\sqrt{u}\right\}_1^2$$

$$= \left(\frac{4\sqrt{2}}{3} + 2\sqrt{2}\right) - \left(\frac{2}{3} + 2\right) = \frac{10\sqrt{2} - 8}{3}.$$

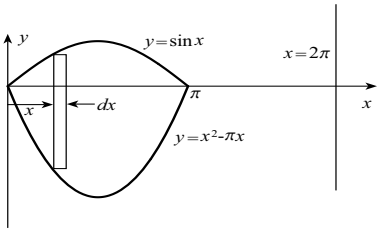


6 **2.** Set up, but do **NOT** evaluate, a definite integral to find the volume of the solid of revolution when the area bounded by the curves

$$y = \sin x, \qquad y = x^2 - \pi x, \quad 0 \le x \le \pi,$$

is rotated around the line $x = 2\pi$.

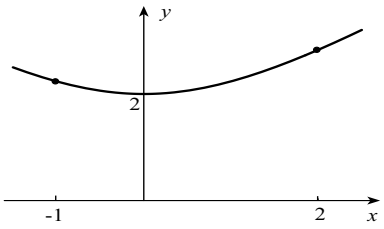
$$V = \int_0^\pi 2\pi(2\pi - x)[\sin x - x^2 + \pi x]\,dx$$



5 **3.** Set up, but do **NOT** evaluate, a definite integral to find the length of the curve $y^2 - x^2 = 4$ between the points $(-1, \sqrt{5})$ and $(2, 2\sqrt{2})$.

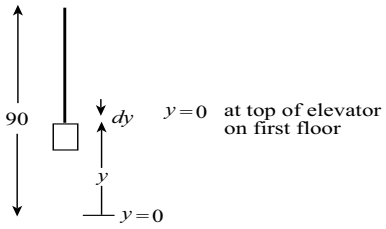
$$L = \int_{-1}^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \int_{-1}^2 \sqrt{1 + \left(\frac{x}{\sqrt{x^2+4}}\right)^2}\,dx$$



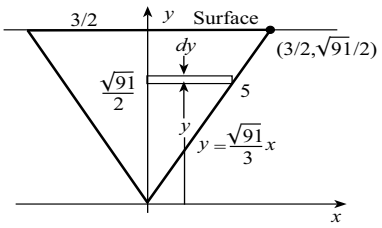
7 **4.** An elevator with mass 5000 kg is sitting on the first floor of a building. The elevator is lifted by a cable with mass 5 kilograms per metre of length. The length of cable from elevator to pulley at the top of the elevator shaft is 90 metres. Set up, but do **NOT** evaluate, a definite integral to find the work done to lift the elevator and cable a total distance of 30 metres from its present position.

$$W = \int_0^{30} 9.81[5000 + 5(90 - y)]\, dy \text{ J}$$



7 **5.** A plate is in the shape of an isosceles triangle with equal sides of length 5 metres and the third side of length 3 metres. It is suspended vertically in water with its shortest side in the surface of the water. Set up, but do **NOT** evaluate, a definite integral to find the force due to the water on one side of the plate.

$$F = 2\int_0^{\sqrt{91}/2} 9.81(1000)\left(\frac{\sqrt{91}}{2} - y\right)\frac{3y}{\sqrt{91}}dy \text{ N}$$



6 **6.** A plate with constant mass per unit area ρ is bounded by the curves

$$x = y^2 - 4, \qquad x + 2y = 4.$$

Set up, but do **NOT** evaluate, a definite integral to find the first moment of the plate about the y-axis.

$$\text{Moment} = \int_{-4}^{2} \rho\left[\frac{(y^2 - 4) + (4 - 2y)}{2}\right][(4 - 2y) - (y^2 - 4)]\, dy$$