Math 1710 Test 2 **March 16, 2010** **60 minutes**

Student Name - **Student Number -**

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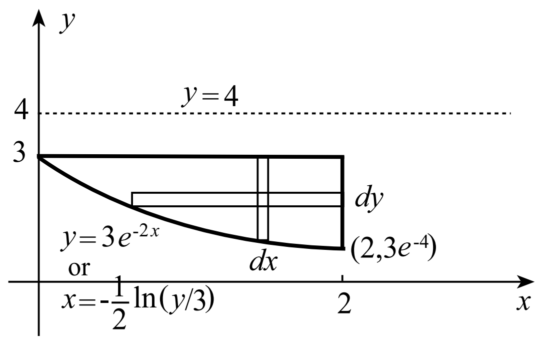
Values

9 **1.** Set up, but do **NOT** evaluate, a definite integral to find the moment of inertia of a uniform plate with constant mass per unit area ρ about the line $y = 4$ if the edges of the plate are the curves

$$y = 3e^{-2x}, \quad y = 3, \quad x = 2.$$



With horizontal rectangles,

$$I = \int_{3e^{-4}}^{3} \rho(y-4)^2 \left[2 + \frac{1}{2}\ln(y/3)\right] dy.$$

The moment of inertia of a vertical rectangle is

$$I_{LSR} = \int_{3e^{-2x}}^{3} \rho(y-4)^2 \, dx \, dy = \rho dx \left\{\frac{1}{3}(y-4)^3\right\}_{3e^{-2x}}^{3} = \frac{\rho}{3}\left[(3-4)^3 - (3e^{-2x}-4)^3\right] dx.$$

For the plate then,

$$I = \int_0^2 \frac{\rho}{3}\left[-1 - (3e^{-2x}-4)^3\right] dx.$$

8 **2.** Assuming that the equation

$$(x-1)^2y+\mathrm{Tan}^{-1}(xy)+y=4$$

defines y implicitly as a function of x, find dy/dx when $x=0$.

Implicit differentiation with respect to x gives

$$2(x-1)y+(x-1)^2\frac{dy}{dx}+\frac{1}{1+x^2y^2}\left(y+x\frac{dy}{dx}\right)+\frac{dy}{dx}=0.$$

We solve this for

$$\frac{dy}{dx}=-\frac{2(x-1)y+\dfrac{y}{1+x^2y^2}}{(x-1)^2+\dfrac{x}{1+x^2y^2}+1}.$$

When $x=0$, the original equation gives

$$(-1)^2y+0+y=4 \quad\Longrightarrow\quad y=2.$$

The derivative at $x=0$ is therefore

$$\frac{dy}{dx}=-\frac{2(-1)(2)+2}{(-1)^2+1}=1.$$

8 **3.** Evaluate the indefinite integral

$$\int \frac{\sqrt{x}}{2+\sqrt{x}} dx.$$

If we set $u = \sqrt{x}$, then $x = u^2$ and $dx = 2u\,du$.

$$\begin{aligned}\int \frac{\sqrt{x}}{2+\sqrt{x}} dx &= \int \frac{u}{2+u}(2u\,du) = 2\int \frac{u^2}{u+2} du \\ &= 2\int \left(u - 2 + \frac{4}{u+2}\right) du = 2\left(\frac{u^2}{2} - 2u + 4\ln|u+2|\right) + C \\ &= x - 4\sqrt{x} + 8\ln\left(\sqrt{x}+2\right) + C.\end{aligned}$$

7 **4.** Evaluate the indefinite integral

$$\int (2x-1)\ln x\,dx.$$

If we set $u = \ln x$ and $dv = (2x-1)\,dx$, then $du = (1/x)\,dx$ and $v = x^2 - x$. Integration by parts gives

$$\begin{aligned}\int (2x-1)\ln x\,dx &= (x^2-x)\ln x - \int (x^2-x)\frac{1}{x}dx \\ &= (x^2-x)\ln x - \int (x-1)\,dx \\ &= (x^2-x)\ln x - \frac{x^2}{2} + x + C.\end{aligned}$$

8 **5.** Evaluate the definite integral

$$\int_0^1 x^3(2x^2+1)^6\,dx.$$

A numerical answer is required, but it need not be simplified.

If we set $u = 2x^2+1$, then $du = 4x\,dx$, and

$$\begin{aligned}
\int_0^1 x^3(2x^2+1)^6\,dx = \int_0^1 x^2(2x^2+1)^6(x\,dx) &= \int_1^3 \left(\frac{u-1}{2}\right)u^6\left(\frac{du}{4}\right)\\
&= \frac{1}{8}\int_1^3 (u^7-u^6)\,du\\
&= \frac{1}{8}\left\{\frac{u^8}{8}-\frac{u^7}{7}\right\}_1^3\\
&= \frac{1}{8}\left[\left(\frac{3^8}{8}-\frac{3^7}{7}\right)-\left(\frac{1}{8}-\frac{1}{7}\right)\right].
\end{aligned}$$