MATH 2130 Test 1 Fall 2024

15 **1.** Find the equation of the plane that contains the point $(1,-1,2)$ and the line

$$x+y-2z=4, \qquad x-y+z=2.$$

If we set $z=0$ in the equations for the line

$$x+y=4, \quad x-y=2, \qquad \text{and these imply that} \qquad x=3, \quad y=1.$$

A point on the line is $Q(3,1,0)$. The vector joining $P(1,-1,2)$ and Q is $\overline{\mathbf{PQ}}=(2,2,-2)$. A vector along the line is

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -2 \\ 1 & -1 & 1 \end{vmatrix} = (-1,-3,-2) \qquad \text{as is} \qquad (1,3,2).$$

A vector perpendicular to the plane is

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 2 & -2 \\ 1 & 3 & 2 \end{vmatrix} = (10,-6,4) \qquad \text{as is} \qquad (5,-3,2).$$

The equation of the plane is

$$5(x-3)-3(y-1)+2z=0 \qquad \Longrightarrow \qquad 5x-3y+2z=12.$$

15 **2.** Find the distance between the lines

$$\ell_1: \quad \begin{aligned} x+2y+z&=4, \\ x-y+z&=1, \end{aligned} \qquad\qquad \ell_2: \quad \begin{aligned} x&=1+2t, \\ y&=-1+t, \\ z&=4t. \end{aligned}$$

A vector along ℓ_1 is $\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & 1 \\ 1 & -1 & 1 \end{vmatrix} = (3,0,-3)$, and one along ℓ_2 is $(2,1,4)$. Since these vectors are not multiples of one another, the lines are not parallel. A vector perpendicular to both lines is $\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & -1 \\ 2 & 1 & 4 \end{vmatrix} = (1,-6,1)$. A unit vector perpendicular to both lines is $\hat{\mathbf{n}}=(1,-6,1)/\sqrt{38}$.
If we set $z=0$ in ℓ_1,

$$x+2y=4, \quad x-y=1,$$

and these imply that $x=2$, $y=1$. Hence, a point on ℓ_1 is $R(2,1,0)$. A point on ℓ_2 is $S(1,-1,0)$. Vector $\overline{RS}=(-1,-2,0)$. The distance between the lines is

$$|\overline{\mathbf{RS}}\cdot\hat{\mathbf{n}}| = \left|(-1,-2,0)\cdot\frac{(1,-6,1)}{\sqrt{38}}\right| = \frac{11}{\sqrt{38}}.$$

10 **3.** Find all unit tangent vectors to the curve

$$y = xz, \quad z - x^2 = 1,$$

at the point $(1, 2, 2)$.

If we set $x = t$, parametric equations for the curve are $x = t$, $y = t(t^2 + 1)$, $z = t^2 + 1$. A tangent vector to the curve at any point on the curve is

$$\mathbf{T}(t) = (1, 3t^2 + 1, 2t).$$

Since $t = 1$ gives the point $(1, 2, 2)$, a tangent vector at the point is $\mathbf{T}(1) = (1, 4, 2)$. Hence, the two unit tangent vectors to the curve at the point are $\pm(1, 4, 2)/\sqrt{21}$.

10 **4.** Find the equation for the projection of the curve

$$z = \sqrt{16 - x^2 - 2y^2}, \quad x + z = 4$$

in the xy-plane. Describe the projection in detail.

The equation of the projection of the curve in the xy-plane is

$$\begin{aligned}\sqrt{16 - x^2 - 2y^2} &= 4 - x \\ 16 - x^2 - 2y^2 &= 16 - 8x + x^2 \\ 2x^2 + 2y^2 - 8x &= 0 \\ x^2 + y^2 - 4x &= 0 \\ (x - 2)^2 + y^2 &= 4\end{aligned}$$

This is a circle centred at $(2, 0)$ with radius 2.