MATH 2130 Quiz 2 2020 **30 minutes**

1. The equation of the tangent plane to the surface $z = x^3y^2 - 3y$ at the point $(1, 1, -2)$ is

(a) $3x - y - z = 0$

(b) $3x - y + z = 0$

(c) $3x - y - z = 4$

(d) $3x + y - 2z = 8$

(e) none of the above

A normal vector to the tangent plane is

$$\nabla(x^3y^2 - 3y - z)_{|(1,1,-2)} = (3x^2y^2, 2x^3y - 3, -1)_{|(1,1,-2)} = (3, -1, -1).$$

The equation of the tangent plane is therefore

$$3(x - 1) - (y - 1) - (z + 2) = 0 \qquad \Longrightarrow \qquad 3x - y - z = 4.$$

2. Equations for the tangent line to the curve

$$2z = 4x^2 - y^2 + 3, \qquad xyz - 2y = -1$$

at the point $(1, -1, 3)$ are:

(a) $x = 1, \quad z - y = 4$

(b) $x = 1 + t, \quad y = -1 + t, \quad z = 3 + t$

(c) $x + y = 0, \quad x + z = 4$

(d) $x = 1, \quad y = -1 - t, \quad z = 3 + t$

(e) None of the above

A normal to the surface $2z = 4x^2 - y^2 + 3$ at $(1, -1, 3)$ is

$$\nabla(4x^2 - y^2 - 3 - 2z)_{|(1,-1,3)} = (8x, -2y, -2)_{|(1,-1,3)} = (8, 2, -2).$$

A normal to the surface $xyz - 2y = -1$ at $(1, -1, 3)$ is

$$\nabla(xyz - 2y + 1)_{|(1,-1,3)} = (yz, xz - 2, xy)_{|(1,-1,3)} = (-3, 1, -1).$$

A tangent vector to the curve at $(1, -1, 3)$ is

$$(4, 1, -1) \times (-3, 1, -1) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 1 & -1 \\ -3 & 1 & -1 \end{vmatrix} = (0, 7, 7).$$

Parametric equations for the tangent line are

$$x = 1, \quad y = -1 + t, \quad z = 3 + t.$$

These imply that $x = 1$ and $z - y = 4$.

3. The rate of change of the function $f(x,y,z) = 2x + 4y^2 - 5z$ with respect to distance along the line $x = 2 + t$, $y = 3 - t$, $z = 4 + 2t$ at the point $(3,2,6)$ in the direction of increasing t is:

(a) -8

(b) $-2\sqrt{6}$

(c) $4\sqrt{6}$

(d) $-4\sqrt{6}$

(e) None of the above

Since

$$\nabla f_{|(3,2,6)} = (2, 8y, -5)_{|(3,2,6)} = (2, 16, -5),$$

and a tangent vector to the curve is $\mathbf{T} = (1, -1, 2)$, the required rate of change is

$$D_{\mathbf{T}} f = (2, 16, -5) \cdot \frac{(1,-1,2)}{\sqrt{6}} = -\frac{24}{\sqrt{6}} = -4\sqrt{6}.$$

4. You are given that $u = f(v)$, $v = g(x,y)$, $x = h(s,t)$, $y = k(t)$. The chain rule for $\partial u/\partial t$ is:

(a) $\dfrac{du}{dv}\left(\dfrac{\partial v}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial v}{\partial y}\dfrac{dy}{dt}\right)$

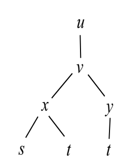
(b) $\dfrac{\partial u}{\partial v}\dfrac{\partial v}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial u}{\partial v}\dfrac{\partial v}{\partial y}\dfrac{\partial y}{\partial t}$

(c) $\dfrac{du}{dv}\dfrac{dv}{dx}\dfrac{dx}{dt} + \dfrac{du}{dv}\dfrac{dv}{dy}\dfrac{dy}{dt}$

(d) $\dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial v}{\partial y}\dfrac{dy}{dt}$

(e) None of the above

The schematic to the right gives
$\dfrac{\partial u}{\partial t} = \dfrac{du}{dv}\dfrac{\partial v}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{du}{dv}\dfrac{\partial v}{\partial y}\dfrac{dy}{dt}.$
This is equivalent to answer (a).



5. The equations

$$F(x,y,z,s,t,u) = 0, \quad G(x,y,z,s,t,u) = 0, \quad H(x,y,z,s,t,u) = 0$$

define x, y, and z as functions of s, t, and u. Which of the following quotients should be used to calculate $\partial y/\partial t$?

(a) $\dfrac{\begin{vmatrix} F_x & F_t & F_z \\ G_x & G_t & G_z \\ H_x & H_t & H_z \end{vmatrix}}{\begin{vmatrix} F_x & F_y & F_z \\ G_x & G_y & G_z \\ H_x & H_y & H_z \end{vmatrix}}$

(b) $\dfrac{\begin{vmatrix} F_x & F_y & F_z \\ G_x & G_y & G_z \\ H_x & H_y & H_z \end{vmatrix}}{\begin{vmatrix} F_x & F_t & F_z \\ G_x & G_t & G_z \\ H_x & H_t & H_z \end{vmatrix}}$

(c) $-\dfrac{\begin{vmatrix} F_x & F_t & F_z \\ G_x & G_t & G_z \\ H_x & H_t & H_z \end{vmatrix}}{\begin{vmatrix} F_x & F_y & F_z \\ G_x & G_y & G_z \\ H_x & H_y & H_z \end{vmatrix}}$

(d) $-\dfrac{\begin{vmatrix} F_x & F_y & F_z \\ G_x & G_y & G_z \\ H_x & H_y & H_z \end{vmatrix}}{\begin{vmatrix} F_x & F_t & F_z \\ G_x & G_t & G_z \\ H_x & H_t & H_z \end{vmatrix}}$

(e) None of the above

$$\frac{\partial y}{\partial t} = -\frac{\begin{vmatrix} F_x & F_t & F_z \\ G_x & G_t & G_z \\ H_x & H_t & H_z \end{vmatrix}}{\begin{vmatrix} F_x & F_y & F_z \\ G_x & G_y & G_z \\ H_x & H_y & H_z \end{vmatrix}}$$