MATH 2130 Quiz 3 2020 **30 minutes**

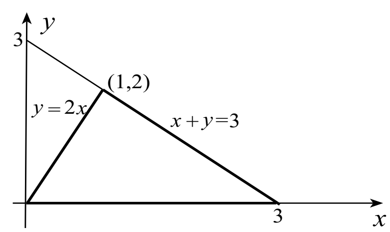
1. The value of the double integral of the function $f(x, y) = xy \sin y$ over the area bounded by the curves $y = 2x$, $x + y = 3$, $y = 0$ is:

(a) Negative

(b) Positive

(c) Zero

(d) There is not enough information to tell whether the value is one of the above.



Since the function $f(x, y) = xy \sin y$ is always positive in the region of integration, the double integral must be positive.

2. Which of the following double iterated integrals yields the volume of the solid of revolution when the area bounded by the curves $y = x^2 - 1$, $y - 2x = 2$ is rotated around the line $x + y = 10$?

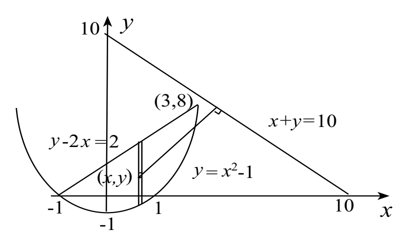
(a) $\displaystyle\int_{-1}^{3}\int_{x^2-1}^{2x+2} \sqrt{2}\pi(x + y - 10)\, dy\, dx$

(b) $\displaystyle\int_{-1}^{3}\int_{x^2-1}^{2x+2} \sqrt{2}\pi x\, dy\, dx$

(c) $\displaystyle\int_{-1}^{3}\int_{-1}^{8} \sqrt{2}\pi(10 - x - y)\, dy\, dx$

(d) $\displaystyle\int_{-1}^{3}\int_{x^2-1}^{2x+2} \sqrt{2}\pi(10 - x - y)\, dy\, dx$

(e) None of the above



The volume is

$$\int_{-1}^{3}\int_{x^2-1}^{2x+2} 2\pi\left(\frac{|x + y - 10|}{\sqrt{2}}\right) dy\, dx = \int_{-1}^{3}\int_{x^2-1}^{2x+2} \sqrt{2}\pi(10 - x - y)\, dy\, dx$$

3. Which of the following double iterated integrals gives a value equal to the double iterated integral

$$\int_{-4}^{2} \int_{0}^{\sqrt{x+4}} x \sin\left(y^2\right) dy\, dx$$

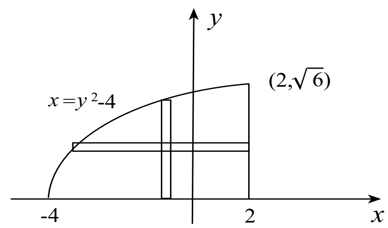
(a) $\displaystyle\int_{0}^{\sqrt{6}} \int_{y^2-4}^{2} x \sin\left(y^2\right) dx\, dy$

(b) $\displaystyle\int_{0}^{\sqrt{6}} \int_{-4}^{2} x \sin\left(y^2\right) dx\, dy$

(c) $\displaystyle\int_{0}^{\sqrt{6}} \int_{y^2+4}^{2} x \sin\left(y^2\right) dx\, dy$

(d) $\displaystyle\int_{0}^{\sqrt{6}} \int_{y^2-4}^{2} y \sin\left(x^2\right) dx\, dy$

(e) None of the above



When we reverse the order of integration, the double iterated integral is $\displaystyle\int_{0}^{\sqrt{6}} \int_{y^2-4}^{2} x \sin\left(y^2\right) dx\, dy$

4. An elliptic plate has edge equation $x^2+2y^2=4$. It is fully submerged in oil (density 980 kilograms per cubic metre) with its upper most point in the surface of the oil. Which of the following double iterated integrals represents the force of the oil on one side of the plate?

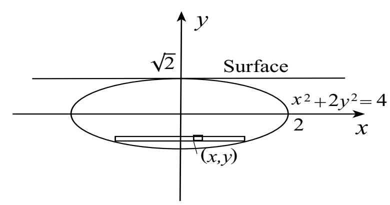
(a) $4\int_0^{\sqrt{2}}\int_0^{\sqrt{4-2y^2}} (9.81)(980)(\sqrt{2}-y)\,dx\,dy$

(b) $\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{4-y^2}}^{\sqrt{4-2y^2}} (9.81)(980)(-y)\,dx\,dy$

(c) $\int_{-\sqrt{2}}^{\sqrt{2}}\int_0^{\sqrt{4-2y^2}} (\sqrt{2}-y)\,dx\,dy$

(d) $\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{4-y^2}}^{\sqrt{4-2y^2}} (9.81)(980)(\sqrt{2}-y)\,dx\,dy$

(e) None of the above



The force is

$$\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} 9.81(980)(\sqrt{2}-y)\,dx\,dy$$

5. A thin plate with constant mass per unit area ρ is bounded by the curves $y=x^3$, $x=\sqrt{2-y}$, $x=0$. Which of the following double iterated integrals gives the moment of inertia of the plate about the line $x+y=1$?

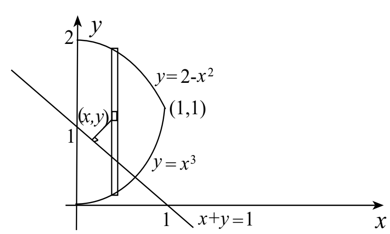
(a) $\int_0^1\int_{x^3}^{2-x^2} \frac{\rho(x+y-1)^2}{2}\,dy\,dx$

(b) $\int_0^1\int_{x^3}^{2-x^2} \frac{\rho(x+y-1)^2}{\sqrt{2}}\,dy\,dx$

(c) $\int_0^1\int_0^2 \frac{\rho(x+y-1)^2}{2}\,dy\,dx$

(d) $\int_0^1\int_{x^3}^{2-x^2} \frac{\rho|x+y-1|}{\sqrt{2}}\,dy\,dx$

(e) None of the above



The moment of inertia is

$$\int_0^1\int_{x^3}^{2-x^2}\left(\frac{|x+y-1|}{\sqrt{2}}\right)^2\rho\,dy\,dx=\int_0^1\int_{x^3}^{2-x^2}\frac{(x+y-1)^2}{2}\rho\,dy\,dx$$