Sample Exam

1. Consider the curve C of intersection of the surfaces

$$z = 6 - \sqrt{x^2 + (y-1)^2}, \qquad y + z = 5.$$

(a) Find the equation of the projection C_{xy} of C onto the xy-plane, simplified as much as possible.

(b) Use part (a) to find parametric equations for C directed so that x decreases along the curve.

(c) Set up, but do **NOT** evaluate, a definite integral for the length of that part of C that joins the points $(0,0,5)$ and $(\sqrt{20},5,0)$.

2. Find the distance between the lines

$$x = t, \quad y = 3t - 1, \quad z = 1 + 2t, \qquad \text{and} \qquad x = 2u + 1, \quad y = 1 - u, \quad z = 4 + 2u.$$

3. Find the rate of change of the function

$$f(x,y,z) = \cos(\pi xy) + x \ln(z^2 + 1),$$

with respect to length s along the curve

$$y = -3x, \qquad z = x^2 - y^2 + 9,$$

directed so that x increases, at the point $(-1,3,1)$.

4. Show that the function $G(x,y) = f(3x - 2y^2) + xy$ satisfies the equation

$$4y\frac{\partial G}{\partial x} + 3\frac{\partial G}{\partial y} = 3x + 4y^2.$$

5. The function

$$f(x,y) = x^4 - 3x^2y^2 + y^4 + x^2 + y^2$$

is known to have five critical points $(0,0)$, $(1,\pm1)$, and $(-1,\pm1)$. It is **NOT** necessary for you to show this. Classify the two critical points $(0,0)$ and $(1,1)$ as yielding relative maxima, relative minima, or saddle points.

6. The function

$$f(x,y) = x^4 - 3x^2y^2 + y^4$$

is known to have critical point $(0,0)$, and the second derivative test fails to determine whether this critical point gives a relative maximum, a relative minimum, or a saddle point. Use whatever method you can devise to perform this classification.

7. Find the maximum value of the function

$$f(x, y) = x^2 - y^2$$

on the region $x^2 + y^2 \leq 1$.

8. Evaluate the double iterated integral

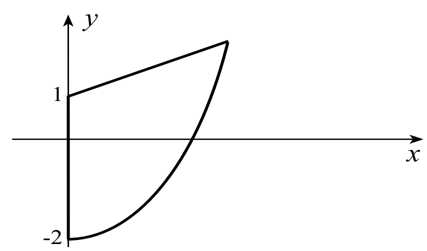
$$\int_0^1 \int_x^1 \sin(\pi y^2)\, dy\, dx.$$

9. Find the area of that part of the "saddle" $z = x^2 - y^2$ cut out by the cylinder $x^2 + y^2 = a^2$, where $a > 0$ is a constant.

10. The region bounded by the curves

$$x = \sqrt{y+2}, \quad x = 2y - 2, \quad x = 0$$

is shown to the right.



(a) Set up, but do **NOT** evaluate, double iterated integral(s) for the volume of the solid of revolution obtained by rotating the region about the line $x = -4$.

(b) If the region represents a thin plate with mass per unit area $\rho(x, y) = x^2 + y^2$, set up, but do **NOT** evaluate double iterated integral(s) for the moment of inertia of the plate about the edge $x = 2y - 2$. You may use the formula

$$\frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

for the distance from a point (x_0, y_0) to a line $Ax + By + C = 0$.

11. Set up, but do **NOT** evaluate, triple iterated integrals to determine the volume of the solid lying below the sphere $x^2 + y^2 + z^2 = 4$ and above the surface $z = \sqrt{x^2 + y^2}$ using:
(a) cylindrical coordinates; and
(b) spherical coordinates.

Answers

1.(a) $y = x^2/4$ (b) $x = -t$, $y = t^2/4$, $z = 5 - t^2/4$ (c) $\displaystyle\int_{-\sqrt{20}}^0 \sqrt{1 + \frac{t^2}{4} + \frac{t^2}{4}}\, dt$

2. $9/\sqrt{117}$ **3.** $(\ln 2 - 16)/\sqrt{266}$ **5.** $(0, 0)$ gives relative minimum; $(1, 1)$ gives saddle point
6. Saddle point **7.** 1 **8.** $1/\pi$ **9.** $\pi[(1 + 4a^2)^{3/2} - 1]/6$

10.(a) $\displaystyle\int_0^2 \int_{x^2-2}^{(x+2)/2} 2\pi(x+4)\, dy\, dx$ (b) $\displaystyle\int_0^2 \int_{x^2-2}^{(x+2)/2} (x^2 + y^2)\left(\frac{x - 2y + 2}{\sqrt{5}}\right)^2 dy\, dx$

11.(a) $\displaystyle 4\int_0^{\pi/2} \int_0^{\sqrt{2}} \int_0^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$ (b) $\displaystyle 4\int_0^{\pi/2} \int_0^{\pi/4} \int_0^2 R^2 \sin\phi\, dR\, d\phi\, d\theta$