MATH 2130 Test 1 October 4, 2022 60 minutes

7 **1.** (a) Find equations for the projection of the curve

$$x^2 + 3y^2 + z^2 = 9, \quad 2x + z = 2$$

in the xz-plane.
(b) What are the equations of the projection in the xy-plane?

(a) Equations for the projection in the xz-plane are

$$2x + z = 2, \quad y = 0.$$

But we only take part of the line. To find which part, we set $y = 0$ and $z = 2 - 2x$ in the equation of the ellipsoid,

$$x^2 + (2 - 2x)^2 = 9 \quad \Longrightarrow \quad 5x^2 - 8x - 5 = 0 \quad \Longrightarrow \quad x = \frac{8 \pm \sqrt{64 + 100}}{10} = \frac{4 \pm \sqrt{41}}{5}.$$

Thus, we take $\dfrac{4 - \sqrt{41}}{5} \le x \le \dfrac{4 + \sqrt{41}}{5}$.
(b) Equations for the projection in the xy-plane are

$$x^2 + 3y^2 + (2 - 2x)^2 = 9, \quad z = 0.$$

12 **2.** Find, if possible, the equation for the plane that contains the lines

$$L_1: \quad \begin{aligned} x &= 3 - 2t, \\ y &= -4 + 6t, \\ z &= 2t; \end{aligned} \qquad \text{and} \qquad L_2: \quad \begin{aligned} 3x + 2y - z &= 5, \\ x - y + z &= 1. \end{aligned}$$

A vector along L_1 is $\mathbf{v}_1 = (-2, 6, 2)$, and one along L_2 is

$$\mathbf{v}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} = (1, -4, -5).$$

Since these vectors are not multiples, the lines are not parallel. To make a plane, they must intersect. To show this, we substitute the parametric equations of L_1 into the second of the equations for L_2,

$$(3 - 2t) - (-4 + 6t) + (2t) = 1 \quad \Longrightarrow \quad -6t = -6 \quad \Longrightarrow \quad t = 1.$$

This gives the point $(1, 2, 2)$, which satisfies the first of equations L_2. A vector normal to the plane is

$$\mathbf{N} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -2 & 6 & 2 \\ 1 & -4 & -5 \end{vmatrix} = (-22, -8, 2),$$

as is $(11, 4, -1)$. The equation of the plane is therefore $11(x - 1) + 4(y - 2) - (z - 2) = 0$.

10 **3.** Find, if possible, point(s) on the curve

$$x = t^2 + 1, \quad y = 3t - 3, \quad z = 1 - t^2$$

where a tangent vector to the curve is parallel to the vector $(1, -3, -1)$.

Since a tangent vector to the curve is $(2t, 3, -2t)$, it will be parallel to $(1, -3, -1)$ if these exist a scalar λ such that

$$(2t, 3, -2t) = \lambda(1, -3, -1).$$

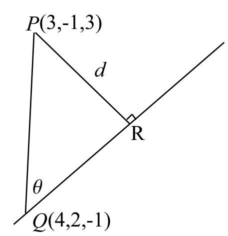
When we equate components,

$$2t = \lambda, \quad 3 = -3\lambda, \quad -2t = -\lambda.$$

These imply thaat $\lambda = -1$ and $t = -1/2$. Thus, there is only one point $(5/4, -9/2, 3/4)$.

11 **4.** Find the distance from the point $(3, -1, 3)$ to the line

$$x = 4 - t, \quad y = 2 + 2t, \quad z = -1 + 3t.$$



$d = |\mathbf{PQ}| \sin\theta = |\mathbf{PQ}||\widehat{\mathbf{QR}}| \sin\theta$
$\quad = |\mathbf{PQ} \times \widehat{\mathbf{QR}}|$
Since a vector along $\mathbf{QR}$ is $(-1, 2, 3)$,

$$\widehat{\mathbf{QR}} = \frac{\pm(-1, 2, 3)}{\sqrt{14}}.$$

Since $Q(4, 2, -1)$ is on the line $\mathbf{PQ} = (1, 3, -4)$.
Thus,

$$d = \left\| \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 3 & -4 \\ -1/\sqrt{14} & 2/\sqrt{14} & 3/\sqrt{14} \end{vmatrix} \right\| = \frac{1}{\sqrt{14}} |(17, 1, 5)| = \frac{\sqrt{315}}{\sqrt{14}}.$$