MATH 2130 Test 1 May 20, 2025 60 minutes

7 **1.** Find the equation for the projection of the curve

$$x = z^2 + y^2, \quad x + 2y + 4z = 1$$

in the yz-plane. Describe the projection in detail.

When we eliminate x between the equations, we obtain

$$z^2 + y^2 = 1 - 2y - 4z \quad \Longrightarrow \quad (y+1)^2 + (z+2)^2 = 6.$$

This is a circle with centre $(-1, -2)$ and radius $\sqrt{6}$.

8 **2.** Set up, but do **NOT** evaluate, a definite integral for the length of the curve

$$x^2 + 2y^2 = 4, \qquad z = x^2 + y^2$$

in the first octant. You need not simplify the integrand.

Parametric equations for the curve are

$$x = 2\cos t, \quad y = \sqrt{2}\sin t, \quad z = 4\cos^2 t + 2\sin^2 t, \qquad 0 \le t \le \pi/2.$$

the length of the curve is given by

$$L = \int_0^{\pi/2} \sqrt{(-2\sin t)^2 + (\sqrt{2}\cos t)^2 + (-8\cos t \sin t + 4\sin t \cos t)^2}\, dt.$$

14 **3.** Find, if possible, the equation for the plane that contains the lines

$$L_1 : \begin{array}{l} x = 4 + t, \\ y = 2 + 5t, \\ z = 1 + 3t; \end{array} \qquad \text{and} \qquad L_2 : \begin{array}{l} x + y - 2z = 3, \\ 2x - y + z = 3. \end{array}$$

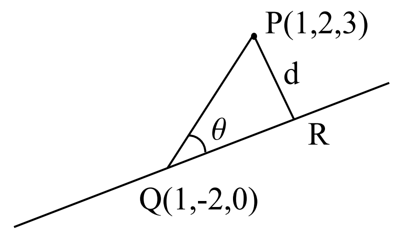
A vector along L_1 is $\mathbf{v}_1 = (1, 5, 3)$. A vector along L_2 is $\mathbf{v}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -2 \\ 2 & -1 & 1 \end{vmatrix} = (-1, -5, -3)$. Since these vectors are multiples, the lines are parallel. A point on L_1 is $P_1 = (4, 2, 1)$, and a point on L_2 is $P_2 = (2, 1, 0)$. Thus, two vectors in the plane are $(1, 5, 3)$ and $\overline{\mathbf{P}_2\mathbf{P}_1} = (2, 1, 1)$. A normal to the plane is

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & 1 \\ 1 & 5 & 3 \end{vmatrix} = (-2, -5, 9), \qquad \text{as is} \quad (2, 5, -9).$$

The equation of the plane is therefore $2(x-2) + 5(y-1) - 9z = 0$, or $2x + 5y - 9z = 9$.

11 **4.** Calculate the distance from the point $(1, 2, 3)$ to the line

$$x = 1 + t, \quad y = -2 + 3t, \quad z = -4t.$$



From the diagram to the right,
$d = |\overline{\mathbf{PQ}}| \sin\theta = |\overline{\mathbf{PQ}}||\widehat{\mathbf{QR}}| \sin\theta$
$\quad = |\overline{\mathbf{PQ}} \times \widehat{\mathbf{QR}}|$
If we choose $Q = (1, -2, 0)$, then $\overline{\mathbf{PQ}} = (0, -4, -3)$.
Since a unit vector along the line is $(1, 3, -4)/\sqrt{26}$,

$$d = \frac{1}{\sqrt{26}} \left\| \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & -4 & -3 \\ 1 & 3 & -4 \end{vmatrix} \right\| = \frac{1}{\sqrt{26}} |(25, -3, 4)| = 5.$$

10 **5.** Find a unit tangent vector to the curve

$$x^2 + y = z, \qquad x + y + z = 4$$

at the point $(2, -1, 3)$.

If we set $x = t$, then parametric equations for the curve are

$$x = t, \quad y = 2 - \frac{t}{2} - \frac{t^2}{2}, \quad z = 2 - \frac{t}{2} + \frac{t^2}{2}.$$

A tangent vector to the curve is $\mathbf{T}(t) = (1, -1/2 - t, -1/2 + t)$. Since $t = 2$ gives the point $\mathbf{T}(2) = (1, -5/2, 3/2)$. Because another tangent vector is $(2, -5, 3)$, a unit tangent vector is $\dfrac{(2, -5, 3)}{\sqrt{38}}$.