MATH 2130 Test 3 March 22, 2012 **60 minutes**

Values

8 **1.** (a) What is the value of the constant C in order that the point $(1,1,1)$ be on the curve

$$x^3y^3 + xy = 2, \qquad 3x + 2y - Cz = 1?$$

(b) Find a unit tangent vector to the curve at the point $(1,1,1)$.

(a) Since $(1,1,1)$ must lie on $3x + 2y - Cz = 1$, it follows that $3(1) + 2(1) - C(1) = 1$, from which $C = 4$.
(b) A vector perpendicular to $x^3y^3 + xy = 2$ at $(1,1,1)$ is

$$\nabla(x^3y^3 + xy - 2)_{|(1,1,1)} = (3x^2y^3 + y, 3x^3y^2 + x, 0)_{|(1,1,1)} = (4,4,0).$$

A vector perpendicular to $3x + 2y - 4z = 1$ is $(3,2,-4)$. A vector tangent to the curve at $(1,1,1)$ is therefore

$$\mathbf{T} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 4 & 0 \\ 3 & 2 & -4 \end{vmatrix} = (-16, 16, -4).$$

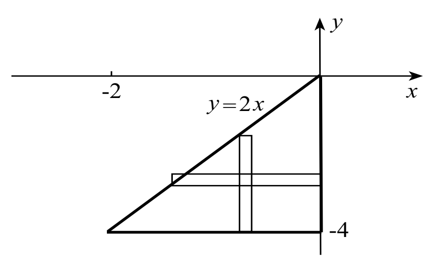
Since $(4,-4,1)$ is also tangent, a unit tangent vector is $\hat{\mathbf{T}} = \dfrac{(4,-4,1)}{\sqrt{33}}$.

12 **2.** Evaluate the double iterated integral

$$\int_{-2}^{0}\int_{-4}^{2x} x\sqrt{x^2+y^2}\,dy\,dx.$$

If we reverse the order of integration

$$\begin{aligned}
\int_{-2}^{0}\int_{-4}^{2x} x\sqrt{x^2+y^2}\,dy\,dx &= \int_{-4}^{0}\int_{y/2}^{0} x\sqrt{x^2+y^2}\,dx\,dy \\
&= \int_{-4}^{0} \left\{\frac{1}{3}(x^2+y^2)^{3/2}\right\}_{y/2}^{0} dy \\
&= \frac{1}{3}\int_{-4}^{0} \left[(y^2)^{3/2} - \left(\frac{y^2}{4} + y^2\right)^{3/2}\right] dy \\
&= \frac{1}{3}\int_{-4}^{0} \left[-y^3 + \left(\frac{5}{4}\right)^{3/2} y^3\right] dy \\
&= \frac{1}{3}\left(\frac{5\sqrt{5}}{8} - 1\right)\left\{\frac{y^4}{4}\right\}_{-4}^{0} \\
&= \frac{8}{3}(8 - 5\sqrt{5}).
\end{aligned}$$



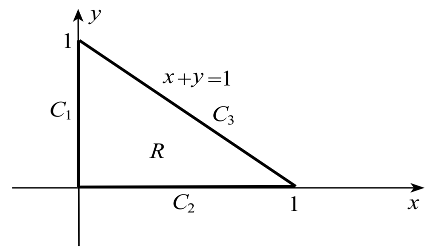
20 **3.** Find the maximum value of the function

$$f(x,y) = xy(2-x-y)$$

on the region $\quad x+y \le 1, \quad x \ge 0, \quad y \ge 0.$

For critical points inside R, we solve
$0 = f_x = 2y - 2xy - y^2 = y(2-2x-y)$,
$0 = f_y = 2x - x^2 - 2xy = x(2-x-2y)$.
Solutions are $(0, 0,$ $(2,0)$, $(0,2)$, and $(2/3, 2/3)$, only the first being in R.

$$f(0,0) = \boxed{0}$$



On C_1 and on C_2, $f(x,y) = \boxed{0}$.
On C_3, $y = 1-x$, and

$$g(x) = f(x, 1-x) = x(1-x)[2-x-(1-x)] = x(1-x), \quad 0 \le x \le 1.$$

For critical points of $g(x)$, we solve

$$g'(x) = 0 = 1 - 2x \qquad \Longrightarrow x = \frac{1}{2}.$$

$$g(0) = \boxed{0}, \quad g(1/2) = \boxed{1/4}, \quad g(1) = 0.$$

The maximum value is 1/4.