MATH 2130 Tutorial 11

1. A triangular plate has sides with lengths 3, 4 and 5 metres. It is submerged vertically in oil with density 950 kilograms per cubic metre. The side of length 3 metres is vertical, the side of length 4 metres is horizontal, and the uppermost vertex is 1 metre below the surface of the oil. Find the force due to oil pressure on each side of the plate.

2. An elliptic plate has major axis of length $2a$ metres and minor axis of length $2b$ metres. Its major axis is horizontal and its minor axes is vertical. It is slowly being lowered into a tank of water. At the instant when only $b/2$ metres of the plate sticks out of the water, set up, but do **NOT** evaluate, a double iterated integral for the force due to the water on each side of the plate.

3. A thin plate with constant mass per unit area ρ has edges defined by the curves

$$x = \sqrt{a^2 - y^2}, \quad y = x, \quad y = 0,$$

where $a > 0$ is a constant. Find the first moment of the plate about the x-axis.

4. A triangular plate has sides of lengths 2, 3 and 3, and constant mass per unit area ρ. Find its moment of inertia about the shorter side.

5. Find the area of that part of the surface $z = xy$ inside the cylinder $x^2 + y^2 = a^2$.

6. Set up, but do **NOT** evaluate, a double iterated integral for the surface area of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

7. Set up, but do **NOT** evaluate, a double iterated integral for the area of the surface $z = 2x^2 + y^2$ bounded by $y = 0$, $x = 0$, and $x + y = 1$.

Answers

1. 1.68×10^5 N

2. $\displaystyle\int_{-b}^{b/2} \int_{-(a/b)\sqrt{b^2-y^2}}^{(a/b)\sqrt{b^2-y^2}} 9810 \left(\frac{b}{2} - y\right) dx\, dy$ N

3. $\rho a^3(\sqrt{2} - 1)/(3\sqrt{2})$

4. $8\sqrt{2}\rho/3$

5. $2\pi[(1 + a^2)^{3/2} - 1]/3$

6. $\displaystyle 8 \int_0^a \int_0^{(b/a)\sqrt{a^2-x^2}} \sqrt{1 + \left(\frac{-cx}{a^2\sqrt{1 - x^2/a^2 - y^2/b^2}}\right)^2 + \left(\frac{-cy}{b^2\sqrt{1 - x^2/a^2 - y^2/b^2}}\right)^2}\, dy\, dx$

7. $\displaystyle\int_0^1 \int_0^{1-x} \sqrt{1 + 16x^2 + 4y^2}\, dy\, dx$