MATH 2130 Tutorial 3

1. The three lines below define a triangle. Find its area.

$$\begin{aligned} &x = -11 + 5s, \qquad && x = 1 + 2u, \qquad && x = -2 + 3t, \\ &y = s, && y = 1 - u, && y = -1 + 2t, \\ &z = -2 + 2s; && z = -2 - 4u; && z = -8 + 6t. \end{aligned}$$

2. The vertices of the triangle in question 1 are three vertices of a parallelogram.What are the possibilities for the fourth vertex?

3. Find the centroid of the triangle in question 1. It is the point of intersection of the three medians of the triangle, or it is the point on one of the medians, two-thirds of the way from the vertex to the midpoint of the opposite side.

4. Find $\mathbf{v}'(3)$ if $\mathbf{v}(t) = t^2\hat{\mathbf{i}} + \mathrm{Sin}^{-1}(t/4)\hat{\mathbf{j}} + \ln(2t+1)\hat{\mathbf{k}}$.

5. If $f(t) = t^2 + 1$ and $\mathbf{v}(t) = e^t\hat{\mathbf{i}} + [t/(t^2+1)^3]\hat{\mathbf{j}} - t\sqrt{t^2+1}\hat{\mathbf{k}}$, evaluate $\int f(t)\mathbf{v}(t)\,dt$.

In questions 6–8, find parametric equations for the curve.

6. $z = 2\sqrt{x^2+y^2}$, $x^2 + y^2 = 3 - z$ from $(1, 0, 2)$ to $(-1, 0, 2)$ so that y is always nonpositive

7. first octant part of $x^2 + z^2 = 4$, $x + y = 1$ directed so that z increases along the curve

8. $z = x^2 + y^2$, $x^2 + y^2 - 4y = 0$ directed clockwise as viewed from a point far up the z-axis

Answers:

1. $(1/2)\sqrt{629}$

2. $(6, 2, 0)$, $(2, 4, 8)$, $(-4, 0, -4)$

3. $(4/3, 2, 4/3)$

4. $6\hat{\mathbf{i}} + \dfrac{1}{\sqrt{7}}\hat{\mathbf{j}} + \dfrac{2}{7}\hat{\mathbf{k}}$

5. $(t^2 - 2t + 3)e^t\hat{\mathbf{i}} - \dfrac{1}{2(t^2+1)}\hat{\mathbf{j}} - \dfrac{1}{5}(t^2+1)^{5/2}\hat{\mathbf{k}} + \overline{C}$

6. $x = \cos t$, $y = -\sin t$, $z = 2$, $0 \le t \le \pi$

7. $x = 2\cos t$, $y = 1 - 2\cos t$, $z = 2\sin t$, $\pi/3 \le t \le \pi/2$

8. $x = 2\cos t$, $y = 2 - 2\sin t$, $z = 8(1 - \sin t)$, $0 \le t \le 2\pi$