MATH 2130 Tutorial 5

In questions 1–7, determine whether the limit exists. If it does not exist, give reasons for its nonexistence.

1. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - xy + y^2}{x^2 + 2y^2}$

2. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$

3. $\displaystyle\lim_{(x,y)\to(2,-3)} \frac{x^2 - 4x - y^2 - 6y - 5}{x^2 - 4x + y^2 + 6y + 13}$

4. $\displaystyle\lim_{(x,y)\to(3,2)} \frac{\sin(2x - 3y)}{2x - 3y}$

5. $\displaystyle\lim_{(x,y)\to(0,1)} \mathrm{Tan}^{-1}\left|\frac{y}{x}\right|$

6. $\displaystyle\lim_{(x,y)\to(0,1)} \mathrm{Tan}^{-1}\left(\frac{y}{x}\right)$

7. $\displaystyle\lim_{(x,y)\to(0,1)} \mathrm{Tan}^{-1}\left(\frac{x}{y}\right)$

8. Show that the function $f(x, y) = 3x^2 + y^2 \cos\left(\frac{2x}{y}\right)$ satisfies the equation

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f(x, y).$$

9. Find all functions $F(x, y, z)$, if there are any, such that

$$\nabla F = (2xy^3 + yze^{xyz})\hat{\mathbf{i}} + (3x^2y^2 + xze^{xyz})\hat{\mathbf{j}} + (xye^{xyz} + y)\hat{\mathbf{k}}.$$

10. The equation

$$F(x, y) = x^3y^2 + 3xy - 4 = 0$$

implicitly defines a curve in the xy-plane. Show that at any point on the curve, the gradient ∇F is perpendicular to the curve.

Answers:

1. Does not exist

2. Does not exist

3. Does not exist

4. 1

5. $\pi/2$

6. Does not exist

7. 0

9. There are none.