MATH 2130 Tutorial 6

1. For what value(s) of the constant b is the function $f(x,y) = e^{bx}\cos 5y$ harmonic in the entire xy-plane?

2. You are told that $z = f(u,v,t)$, $u = g(x,y,t)$, $v = h(x,y,t)$, and $y = k(t)$. What is the chain rule for $\left.\frac{\partial z}{\partial t}\right)_x$?

3. If $u = f(v)$, $v = g(x,y,z)$, $x = h(s,t)$, $y = k(s,t)$, and $z = m(t)$, find the chain rule for $\left.\frac{\partial u}{\partial t}\right)_s$.

4. If $f(s)$ and $g(t)$ are differentiable functions, show that $\nabla f(x^2 - y^2)\cdot\nabla g(xy) = 0$.

5. If $z = x^2 + y^2$, $x = u\cos v$, and $y = u\sin v$, find and simplify $\left.\frac{\partial^2 z}{\partial v^2}\right)_u$.

6. If $f(v)$ is differentiable, show that $u(x,y) = x^3 f(x/y)$ satisfies the equation

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u.$$

7. If $z = f(u,v)$, $u = g(x,y)$, and $v = h(x,y)$, find the chain rule for $\left.\frac{\partial^2 z}{\partial x^2}\right)_y$.

Answers:

1. ± 5

2. $\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial t} + \frac{\partial z}{\partial t}$

3. $\frac{du}{dv}\frac{\partial v}{\partial x}\frac{\partial x}{\partial t} + \frac{du}{dv}\frac{\partial v}{\partial y}\frac{\partial y}{\partial t} + \frac{du}{dv}\frac{\partial v}{\partial z}\frac{dz}{dt}$

5. 0

7. $\frac{\partial^2 z}{\partial u^2}\left(\frac{\partial u}{\partial x}\right)^2 + 2\frac{\partial^2 z}{\partial u\partial v}\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial^2 z}{\partial v^2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{\partial z}{\partial u}\frac{\partial^2 u}{\partial x^2} + \frac{\partial z}{\partial v}\frac{\partial^2 v}{\partial x^2}$