

## MATH 2130 Tutorial 7

1. The equations

$$x^2 + y + 3s^2 + s = 2t - 1, \quad y^2 - x^4 + 2st + 7 = 6s^2t^2,$$

define  $s$  and  $t$  as functions of  $x$  and  $y$ . Find  $\partial s/\partial x$  when  $s = 0$  and  $t = 1$ . Assume that  $x > 0$ .

2. The equations

$$x^3y^2 + uv = x + y + 2, \quad xy - y(u^2 + v^2) = 3u + 3,$$

define  $u$  and  $v$  as functions of  $x$  and  $y$ . Find  $\partial u/\partial y$  when  $x = 1$  and  $y = 0$ .

3. The equations

$$x = r \sin \phi \cos \theta, \quad y = r \sin \phi \sin \theta, \quad z = r \cos \phi,$$

define  $r$ ,  $\phi$ , and  $\theta$  as functions of  $x$ ,  $y$ , and  $z$ . Find  $\partial \phi/\partial y$ .

4. The function  $f(x, y, z) = x^2y + z^3$  is defined at every point on the curve

$$x(y + z) = 3, \quad y - z = 4,$$

directed so that  $y$  increases along the curve. What is the rate of change of the function with respect to distance travelled along the curve at the point  $(1, 7/2 - 1/2)$ ?

5. In what direction(s) is the rate of change of the function  $f(x, y) = x^2y - xy^2$  with respect to distance equal to (a)  $-1$ , (b)  $4$  at the point  $(1, 1)$ ?
6. At the point  $(1, 2, -3)$ , a vector  $\mathbf{v}$  makes an angle of  $\pi/3$  radians with the gradient of the function  $f(x, y, z) = x^2yz - 3xy^3$ . Find the rate of change of  $f(x, y, z)$  in direction  $\mathbf{v}$ .

### Answers:

- 16
- $-3$
- $r^{-1} \cos \phi \sin \theta$
- $-35/(4\sqrt{22})$
- (a)  $\hat{\mathbf{j}}$ ,  $-\hat{\mathbf{i}}$  (b) None
- $\sqrt{2821}/2$