

MATH 2130 Tutorial 8

1. Find the rate of change of the function $f(x, y, z) = \sin(xy) - z^3$ at the point $(2, 0, 3)$ in the direction of the upward normal to the surface $xz^2 - x^2z = 6$.

2. Find equations for the tangent line to the curve

$$xyz + z^3 = 24, \quad x^3y^2z + y^3 = 4x - 2$$

at the point $(1, -1, 3)$.

3. Find the acute angle between the normal to the surface $x + z = 3$ and the tangent line to the curve $xy^3z + z^3 = 6$, $xy + yz = -3$ at their point of intersection.

4. Find all critical points for the function $f(x, y) = x^3y^3 - x^2y^2 + 6$.

5. Find all critical points for the function $f(x, y) = x^3y^2 - xy + 3y$.

Answers:

1. $-216/\sqrt{73}$

2. $x = 1 + 81t, y = -1 + 133t, z = 3 - 6t$

3. $\cos^{-1}\left(\frac{3\sqrt{5}}{10}\right)$

4. Every point on x -axis, every point on y -axis, and every point on the curve $y = 2/(3x)$.

5. $(3, 0), (9, 1/243)$