MATH 2130 Tutorial 9

In questions 1–4, find and classify all critical points of the function as giving relative maxima, relative minima, saddle points, or none of these.

1. $f(x, y) = x^3 + xy + y^3$

2. $f(x, y) = x^3 - xy^2 + 3xy$

3. $f(x, y) = x^4 - 3x^2y^2 + y^4$

4. $f(x, y) = y^2 + |x - 1|$

5. Find the maximum and minimum values of the function $f(x, y) = x^2 - y^2$ on the region $x^2 + y^2 \leq 1$.

6. Find the maximum value of the function $f(x, y) = xy(3 - x - 2y)$ on the triangle R bounded by the positive x- and y-axes and the line $x + y = 1$.

7. Find the maximum value of the function $f(x, y) = x^2 - y^2 + 2x + 9y/2$ considering only points inside and on the boundary of the region surrounded by the curves

$$x = 1 - y^2, \quad x = 0.$$

Answers:

1. $(0, 0)$ gives a saddle point; $(-1/3, -1/3)$ gives a relative maximum

2. $(0, 0)$ gives a saddle point; $(0, 3)$ also gives a saddle point

3. $(0, 0)$ gives a saddle point

4. $(1, 0)$ gives a relative minimum. Points $(1, y)$, $y \neq 0$ give none of these.

5. $1, -1$

6. $2\sqrt{3}/9$

7. $65/16$