THE UNIVERSITY OF MANITOBA

DATE: December 17, 2011 **FINAL EXAMINATION**

DEPARTMENT & NO: MATH2132 **TIME**: 3 hours

EXAMINATION: Engineering Mathematical Analysis 2 **EXAMINER**: M. Despic, D. Trim

INSTRUCTIONS:

1. No aids permitted.

2. Attempt all questions.

3. If insufficient space is provided for a solution to a problem, continue your work on the back of the previous page.

4. Check that your examination booklet contains pages numbered from 1 to 12.

5. Fill in the information requested below.

Student Name (Print): ______________________

Student Signature: ______________________

Student Number: ______________________

Seat Number: ______________________

Question	Maximum Mark	Assigned Mark	Question	Maximum Mark	Assigned Mark
1			7		
2			8		
3			9		
4			10		
5			11		
6			12		
Total			Total		

Examination Total /100

10 **1.** For what value of the constant a will the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n^2+1}{a^n}(x-1)^{2n}$$

be equal to 5?

13 **2.** Find the Taylor series about $x = 3$ for the function

$$f(x) = (x-3)\ln(x+1).$$

Use a method that guarantees that the series converges to $f(x)$. Express your answer in sigma notation, simplified as much as possible. Determine the open interval of convergence for the series.

7 **3.** Find a maximum possible error for all x in the interval $0 \leq x \leq 3$ if the series

$$\sum_{n=1}^{\infty} \frac{n(-1)^{n+1}}{4^n}(x-1)^n$$

is truncated after its 10^{th} term. Justify your answer.

6 **4.** You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = 3 - 5x + 4\cos x + x^3 e^{-2x}$$

are $m = 0,\ 3 \pm i,\ 3 \pm i,\ -2,\ -2,\ -2,\ \pm\sqrt{5}$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. Do **NOT** find the coefficients, just the form of the particular solution.

9 **5.** Find a one-parameter family of solution for the differential equation

$$y\frac{dy}{dx} = (y+2)(\sin 3x - x).$$

Are there any singular solutions of your family?

15 **6.** (a) A mass of 2 kilograms is suspended from a spring with constant 50 newtons per metre. At time $t = 0$, it is pulled 10 centimetres above its equilibrium position and given velocity 4 metres per second downward. During its subsequent motion, it is also subjected to a damping force that (in newtons) is equal to 20 times its velocity (in metres per second). Determine the maximum distance from its equilibrium position that the mass ever achieves.
(b) If damping is removed, and an additional force $4\cos\omega t$ acts on the mass, what value of ω causes resonance?

12 **7.** (a) Find the Laplace transform for the function in the figure below.

(b) Find the inverse Laplace transform for

$$F(s) = \frac{se^{-2s}}{s^2 + 4s + 7}.$$

10 **8.** Find a general solution of the following initial value problem where $f(t)$ is some unspecified function of time,

$$y'' + 4y' - 5y = f(t), \quad y(0) = 1, \quad y'(0) = 0.$$

15 **9.** A mass of 1 kilogram is suspended from a spring with constant 6 newtons per metre. At time $t = 0$, it is released from 5 centimetres below its equilibrium position. During its subsequent motion, it is subjected to a constant force of 4 newtons upward, and at time $t = 5$ seconds, it is struck upward with an instantaneous force of 3 newtons. Find its position as a function of time.

3 **10.** Use the definition of the Laplace transform to prove that when $f(t)$ has a Laplace transform $F(s)$, then

$$\mathcal{L}\{tf(t)\} = -F'(s).$$

The following table of Laplace transforms may be used without proof.

$f(t)$		$F(s) = \mathcal{L}\{f(t)\}$
$t^n \quad (n = 0, 1, 2, \ldots)$	$\leftrightarrow$	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\leftrightarrow$	$\dfrac{1}{s-a}$
$\sin at$	$\leftrightarrow$	$\dfrac{a}{s^2+a^2}$
$\cos at$	$\leftrightarrow$	$\dfrac{s}{s^2+a^2}$
$h(t-a)$	$\leftrightarrow$	$\dfrac{e^{-as}}{s}$
$\delta(t-a)$	$\leftrightarrow$	e^{-as}
$e^{at}f(t)$	$\leftrightarrow$	$F(s-a)$
$f(t)h(t-a)$	$\rightarrow$	$e^{-as}\mathcal{L}\{f(t+a)\}$
$f(t-a)h(t-a)$	$\leftarrow$	$e^{-as}F(s)$
$p-$ periodic $f(t)$	$\rightarrow$	$\dfrac{1}{1-e^{-ps}}\displaystyle\int_0^p e^{-st}f(t)\,dt$
$\displaystyle\int_0^t f(u)g(t-u)\,du$	$\leftarrow$	$F(s)G(s)$
$f'(t)$	$\rightarrow$	$sF(s) - f(0)$
$f''(t)$	$\rightarrow$	$s^2F(s) - sf(0) - f'(0)$