THE UNIVERSITY OF MANITOBA

DATE: June 18, 2011 **FINAL EXAMINATION**

DEPARTMENT & COURSE NO: MATH2132 **TIME**: 3 hours

EXAMINATION: Engineering Mathematical Analysis 2 **EXAMINER**: D. Trim

10 **1.** Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n2^n}(x-1)^n.$$

8 **2.** Find the Taylor series about $x = 4$ for the function

$$f(x) = \frac{1}{(x+1)^2}.$$

Use a method that guarantees that the series converges to $f(x)$. Express your answer in sigma notation, simplified as much as possible. Determine the open interval of convergence for the series.

12 **3.** (a) Find the Maclaurin series for the function

$$f(x) = x^4 \ln(x+2).$$

Express your answer in sigma notation simplified as much as possible. Include its radius of convergence.

(b) Use the series in part (a) to find $f^{(10)}(0)$.

12 **4.** Find a general solution for the differential equation

$$y'''' + 4y'' + 4y = x - 3\sin 3x.$$

6 **5.** You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = x^3 - x + 2\sin 5x + e^{3x}$$

are $m = 0, 0, \pm 5i, 3 \pm 4i, 6$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. Do **NOT** find the coefficients, just the form of the particular solution.

5 **6.** Find an integrating factor for the differential equation

$$(x+1)\frac{dy}{dx} + xy = \cos 2x, \quad x > 0.$$

Simplify your result as much as possible. Do **NOT** solve the differential equation.

7 **7.** Two substances A and B react to form a third substance C in such a way that 2 grams of A react with 3 grams of B to produce 5 grams of C. The rate at which C is formed is proportional to the product of the amounts of A and B still present in the mixture. Set up an initial-value problem (differential equation plus initial condition) for the amount $C(t)$ of C present in the mixture as a function of time t when the original amounts of A and B brought together at time $t = 0$ are 20 grams and 10 grams, respectively.

8 **8.** Find the Laplace transform for the function $e^{-3t} \sin 2t \, h(t - \pi)$.

4 **9.** Can the function $F(s) = \dfrac{s^2 e^s}{(s^2+1)(e^s+1)}$ be the Laplace transform of a piecewise continuous function of exponential order? Explain.

8 **10.** Find the inverse Laplace transform for the function $F(s) = \dfrac{e^{-2s}(1-e^s)}{s^3+2s}$.

8 **11.** A mass of 1 kilogram is suspended from a spring with constant 50 newtons per metre. At time $t = 0$, it is at its equilibrium position and is given velocity 2 metres per second downward. During its subsequent motion, it is also subjected to air resistance that (in newtons) is equal to 3/2 times its velocity (in metres per second). Use Laplace transforms to find the position of the mass as a function of time.

12 **12.** Find an integral representation for the solution of the initial-value problem

$$y'' - 2y' - 3y = f(t), \qquad y(0) = 1, \quad y'(0) = 0,$$

where $f(t)$ is some unspecified function.

Answers

1. $-1 < x \le 3$ **2.** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{5^{n+2}}(x-4)^n$, $-1 < x < 9$

3.(a) $\displaystyle(\ln 2)x^4 + \sum_{n=5}^{\infty} \frac{(-1)^{n+1}}{2^{n-4}(n-4)} x^n$, 2 (b) $\dfrac{-10!}{6(2^6)}$

4. $y(x) = (C_1 + C_2 x)\cos\sqrt{2}x + (C_3 + C_4 x)\sin\sqrt{2}x + \dfrac{x}{4} - \dfrac{3}{49}\sin 3x$

5. $y_p(x) = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex\sin 5x + Fx\cos 5x + Ge^{3x}$ **6.** $e^x/(x+1)$

7. $\dfrac{dC}{dt} = k\left(20 - \dfrac{2C}{5}\right)\left(10 - \dfrac{3C}{5}\right)$, $C(0) = 0$ **8.** $e^{-\pi(s+3)}\left[\dfrac{2}{(s+3)^2+4}\right]$ **9.** No

10. $\left[\dfrac{1}{2} - \dfrac{1}{2}\cos\sqrt{2}(t-2)\right]h(t-2) - \left[\dfrac{1}{2} - \dfrac{1}{2}\cos\sqrt{2}(t-1)\right]h(t-1)$

11. $x(t) = -\dfrac{8}{\sqrt{791}}e^{-3t/4}\sin\dfrac{\sqrt{791}t}{4}$ m

12. $y(t) = \displaystyle\int_0^t \left[\frac{1}{4}e^{3(t-u)} - \frac{1}{4}e^{-(t-u)}\right] f(u)\,du + \frac{1}{4}e^{3t} + \frac{3}{4}e^{-t}$