THE UNIVERSITY OF MANITOBA

DATE: June 16, 2012 **FINAL EXAMINATION**

DEPARTMENT & COURSE NO: MATH2132 **TIME**: 3 hours

EXAMINATION: Engineering Mathematical Analysis 2 **EXAMINER**: D. Trim

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INSTRUCTIONS:

1. No aids permitted.

2. Attempt all questions.

3. If insufficient space is provided for a solution to a problem, continue your work on the back of the previous page.

4. Check that your examination booklet contains pages numbered from 1 to 12.

5. Fill in the information requested below.

Student Name (Print): ____________________

Student Signature: ____________________

Student Number: ____________________

Seat Number: ____________________

Question	Maximum Mark	Assigned Mark	Question	Maximum Mark	Assigned Mark
1	10		7	7	
2	14		8	9	
3	6		9	9	
4	15		10	8	
5	6		11	10	
6	6				
Total	57		Total	43	

Examination Total /100

10 **1.** Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(-1)^n n}{4^{n+1}}(x-1)^{2n}.$$

14 **2.** Find the Maclaurin series for the function

$$f(x) = \frac{x}{x^2 - x - 2}.$$

Use a method that guarantees that the series converges to $f(x)$. Express your answer in sigma notation, simplified as much as possible. Determine the interval of convergence for the series.

6 **3.** Find a maximum possible error when the function e^{-3x} is approximated by the first three terms in its Maclaurin series on the interval $0 \leq x \leq 0.2$.

15 **4.** Find a general solution for the differential equation

$$3y''' + 2y'' + 2y' - y = x - e^{-2x}.$$

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6 **5.** You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = 2xe^{4x} + x^3 - 2 + 3e^{2x}\cos 5x$$

are $m = 0,\ 2 \pm i,\ 2 \pm i,\ \pm 3,\ 4$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. Do **NOT** find the coefficients, just the form of the particular solution.

6 **6.** When a substance such as glucose is administered intravenously into the bloodstream, it is used up by the body at a rate proportional to the amount present at that time. If it is added at a variable rate $R(t)$, where t is time, and A_0 is the amount in the bloodstream when the intravenous feeding begins, set up, but **DO NOT SOLVE**, an initial value problem for the amount of glucose in the bloodstream at any time. Is the differential equation separable?

7 **7.** Find an implicit definition for the solution of the initial value problem

$$y^2 \frac{dy}{dx} = (x+1)(y^3+1), \qquad y(0) = 1.$$

9 **8.** Find the Laplace transform for the function

$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ 4 - t, & 2 < t \le 4 \end{cases} \qquad f(t+4) = f(t).$$

Simplify the transform as much as possible.

9 **9.** Find the inverse Laplace transform for the function

$$F(s) = \frac{e^{-2s}(3s^2+2)}{s^3 - s^2 + 2}.$$

8 **10.** A mass of 1 kilogram is suspended from a spring with constant 400 newtons per metre. At time $t = 0$, it is at its equilibrium position and is given velocity 2 metres per second upward. During its subsequent motion, it is also subjected to a damping force that (in newtons) is equal to 40 times its velocity (in metres per second). Use Laplace transforms to find the position of the mass as a function of time.

10 **11.** Solve the initial value problem

$$y'' - 3y' - 4y = 3\delta(t-2), \qquad y(0) = 0, \quad y'(0) = 1.$$

The following table of Laplace transforms may be used without proof.

$f(t)$		$F(s) = \mathcal{L}\{f(t)\}$
$t^n \quad (n = 0, 1, 2, \ldots)$	$\leftrightarrow$	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\leftrightarrow$	$\dfrac{1}{s-a}$
$\sin at$	$\leftrightarrow$	$\dfrac{a}{s^2+a^2}$
$\cos at$	$\leftrightarrow$	$\dfrac{s}{s^2+a^2}$
$h(t-a)$	$\leftrightarrow$	$\dfrac{e^{-as}}{s}$
$\delta(t-a)$	$\leftrightarrow$	e^{-as}
$e^{at}f(t)$	$\leftrightarrow$	$F(s-a)$
$f(t)h(t-a)$	$\rightarrow$	$e^{-as}\mathcal{L}\{f(t+a)\}$
$f(t-a)h(t-a)$	$\leftarrow$	$e^{-as}F(s)$
$p-$ periodic $f(t)$	$\rightarrow$	$\dfrac{1}{1-e^{-ps}}\displaystyle\int_0^p e^{-st}f(t)\,dt$
$\displaystyle\int_0^t f(u)g(t-u)\,du$	$\leftarrow$	$F(s)G(s)$
$f'(t)$	$\rightarrow$	$sF(s) - f(0)$
$f''(t)$	$\rightarrow$	$s^2F(s) - sf(0) - f'(0)$