THE UNIVERSITY OF MANITOBA

DATE: April 23, 2014 **FINAL EXAMINATION**

DEPARTMENT & COURSE NO: MATH2132 **TIME**: 3 hours

EXAMINATION: Engineering Mathematical Analysis 2 **EXAMINER**: D. Trim

10 **1.** Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{1}{ne^{an}}(x-1)^n,$$

where $a > 0$ is a constant.

8 **2.** Find the Taylor series about $x = -2$ for the function $(x+2)^3 e^{2(x+4)}$. Express your answer in sigma notation simplified as much as possible. Include its open interval of convergence. You must use a method that guarantees that the series converges to the function.

6 **3.** Use a Maclaurin or Taylor series to evaluate

$$\lim_{x \to 0} \frac{(1-\cos 3x)^3}{x^6}.$$

6 **4.** Determine whether the following series converges or diverges. Justify your conclusion.

$$\sum_{n=1}^{\infty} \frac{n^2 \cos n\pi}{2^n}.$$

10 **5.** Find a one-parameter family of solutions of the differential equation

$$(x+1)\frac{dy}{dx} = (x+1)^{3/2} + 2y.$$

Is your solution a general solution? Explain. On what interval(s) would you claim that your solution is valid?

6 **6.** When chemicals A and B are brought together, chemical A turns into chemical B in such a way that 1 unit of A combines with 1 unit of B to produce 2 units of B. If the rate at which B is formed is proportional to the product of the amounts of A and B in the mixture, find an initial-value problem for the amount of B in the mixture as a function of time t. Assume that the process begins with 10 units of A and 20 units of B. Make **NO ATTEMPT** to solve the problem.

15 **7.** Find a general solution for the differential equation

$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = e^{2x} - x.$$

10 **8.** Find the Laplace transforms for the following functions. Do **NOT** simplify your answers.

(a) $f(t) = 3t^2,\ 0 < t < 2, \quad f(t+2) = f(t)$ (b) $f(t) = t^2 e^{-3t} h(t-4)$

14 **9.** Find inverse Laplace transforms for the following functions:

(a) $F(s) = \dfrac{s+2}{s^3 + 2s^2 + 3s}$ (b) $F(s) = \dfrac{s+1}{s^3(1+e^{-3s})}$

15 **10.** A mass of 2 kilograms is suspended from a spring with constant 20 Newtons per metre. It is set into vertical motion by giving it an upward velocity of 1 metre per second from its equilibrium position. During its subsequent motion, the mass is acted on by a damping force that (in Newtons) is equal to 12 times the velocity (in metres per second). After 2 second, the mass is hit upwards with an impulse force of 5 Newtons. Find its position as a function of time. Does that mass pass through its equilibrium position before the hit at 2 seconds? Justify your answer.

Answers

1. $1 - e^a \le x < 1 + e^a$ **2.** $\displaystyle\sum_{n=3}^{\infty} \frac{e^4 2^{n-3}}{(n-3)!}(x+2)^n,\ -\infty < x < \infty$ **3.** $9^3/8$ **4.** Converges

5. $y(x) = -2(x+1)^{3/2} + C(x+1)^2$, General, $x > -1$ **6.** $\dfrac{dB}{dt} = kB[10 - (B-20)],\ B(0) = 20$

7. $y(x) = (C_1 + C_2)\cos x + (C_3 + C_4 x)\sin x + e^{2x}/25 - x$

8.(a) $\dfrac{3}{1-e^{-2s}}\left[\dfrac{2}{s^3} - e^{-2s}\left(\dfrac{2}{s^3} + \dfrac{4}{s^2} + \dfrac{4}{s}\right)\right]$ (b) $e^{-4(s+3)}\left[\dfrac{2}{(s+3)^3} + \dfrac{8}{(s+3)^2} + \dfrac{16}{s+3}\right]$

9.(a) $\dfrac{2}{3} - \dfrac{1}{3}e^{-t}\left(2\cos\sqrt{2}t - \dfrac{1}{\sqrt{2}}\sin\sqrt{2}t\right)$ (b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\left[(t-3n) + \frac{1}{2}(t-3n)^2\right]h(t-3n)$

10. $x(t) = \dfrac{5}{2}e^{-3(t-2)}\sin(t-2)\,h(t-2) + e^{-3t}\sin t$ m No