MATH 2132 Tutorial 10

1. A 500-gram mass is placed on a table and attached to a spring with constant 20 newtons per metre. The other end of the spring is attached to a wall. The mass is pushed 5 centimetres so as to compress the spring, and then released. The coefficient of kinetic friction between the mass and table is $\mu = 0.1$. Find where the mass stops moving for the first time. Does it move from this position? Take the coefficient of static friction to be 0.2.

2. (a) A 2-kilogram mass is suspended from a spring with constant 1000 newtons per metre. A force $2\sin\omega t$ newtons initiates motion at time $t = 0$, and continues to act on the mass. Find the position of the mass as a function of time when resonance does not occur.
(b) What value of ω causes resonance?

3. Repeat part (a) of problem 2 if a damping force proportional to velocity with $\beta = 10$ acts on the mass.

Answers:

1. With spring compressed $(10 - g)/200$ m. No

2.(a) $\dfrac{-\omega}{10\sqrt{5}(500-\omega^2)}\sin 10\sqrt{5}t + \dfrac{1}{500-\omega^2}\sin\omega t$ (b) $\omega = 10\sqrt{5}$

3. $e^{-5t/2}\left[\dfrac{5\omega}{(500-\omega^2)^2+25\omega^2}\cos\dfrac{5\sqrt{79}t}{2} + \dfrac{\omega(2\omega^2-975)}{5\sqrt{79}[(500-\omega^2)^2+25\omega^2]}\sin\dfrac{5\sqrt{79}t}{2}\right]$
$\dfrac{500-\omega^2}{(500-\omega^2)^2+25\omega^2}\sin\omega t - \dfrac{5\omega}{(500-\omega^2)^2+25\omega^2}\cos\omega t$