MATH 2132 Tutorial 3

1. (a) Find the first five Taylor polynomials for the function $\cos 2x$ about $x = 0$.
(b) Show that the Maclaurin series for $\cos 2x$ converges to $\cos 2x$ for all x.

2. Find the Taylor series about $x = 1$ for the function $f(x) = 1/(x-2)^2$. Express your answer in sigma notation, simplified as much as possible.

3. Find the Maclaurin series for the function $f(x) = 1/(8+3x)^{1/3}$. Express your answer in sigma notation, simplified as much as possible.

In Problems 4–7, find the open interval of convergence for the power series.

4. $\displaystyle\sum_{n=3}^{\infty} \frac{2^n}{n3^{n+1}} x^n$

5. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} x^n$

6. $\displaystyle\sum_{n=0}^{\infty} \frac{(n+5)^4}{3^n} x^n$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{2 \cdot 5 \cdot 8 \cdots (3n+2)} (2x)^n$

8. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{2n}} (x-1)^n$. What is its interval of convergence?

9. The radius of convergence of a power series is nonzero and finite. The interval of convergence is the same as the open interval of convergence. What can you conclude?

Answers:

1.(a) $1,\ 1,\ 1-2x^2,\ 1-2x^2,\ 1-2x^2+2x^4/3$ **2.** $\displaystyle\sum_{n=0}^{\infty} (n+1)(x-1)^n$

3. $\displaystyle\frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n [1 \cdot 4 \cdot 7 \cdots (3n-2)]}{2^{3n+1} n!} x^n$ **4.** $|x| < 3/2$ **5.** $-\infty < x < \infty$ **6.** $-3 < x < 3$

7. $-3/4 < x < 3/4$ **8.** $(1-x)/(x+3)$, $-3 < x < 5$

9. Series does not converge at the end points of the open interval of convergence.