MATH 2132 Tutorial 4

1. Find the open interval of convergence for the power series:

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{n3^n}{n^2+1} x^{2n+3}$ (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} \sqrt{\frac{2n+3}{n+6}} \ln(n+6)(x+2)^n$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{(n!)^3}{(3n)!}(3x-1)^n$

2. Find the interval of convergence for the power series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(3n)4^n}{n+1}(x+1)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{5^n}(2x-3)^n$

3. (a) Find the Maclaurin series for the function $f(x) = x/(4x+1)$. Express your answer in sigma notation, simplified as much as possible What is the interval of convergence of the series?

(b) Repeat part (a), but find the Taylor series about $x = 1$.

4. Find the Taylor series for $\sin x$ about $x = 1$. What is the radius of convergence of the series?

5. Find the Maclaurin series for $\sin^2 2x$. What is the interval of convergence of the series?

6. Find the Taylor series about $x = 5$ for the function $\ln(3+x)$. What is its open interval of convergence?

7. Find the Maclaurin series for the function $1/(4+3x)^2$. What is its interval of convergence?

Answers:

1.(a) $-1/\sqrt{3} < x < 1/\sqrt{3}$ (b) $-3 < x < -1$ (c) $-26/3 < x < 28/3$

2.(a) $-5/4 < x < -3/4$ (b) $2/3 < x < 7/3$

3.(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} 4^{n-1} x^n$, $-1/4 < x < 1/4$

(b) $\displaystyle\frac{1}{5} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4^{n-1}}{5^{n+1}}(x-1)^n$, $-1/4 < x < 9/4$

4. $\displaystyle\cos 1 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(x-1)^{2n+1} + \sin 1 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x-1)^{2n}$, $R = \infty$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{4n-1}}{(2n)!} x^{2n}$, $-\infty < x < \infty$

6. $\displaystyle\ln 8 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n8^n}(x-5)^n$, $-3 < x < 13$

7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^n (n+1)}{4^{n+2}} x^n$, $-4/3 < x < 4/3$