MATH 2132 Tutorial 5

1. Find the Maclaurin series for the function $\text{Tan}^{-1}(2x^2)$. Express your answer in sigma notation, simplified as much as possible. What is the open interval of convergence of the series?

2. Find the Taylor series for $1/\sqrt{10 - 3x}$ about $x = 2$. Express your answer in sigma notation, simplified as much as possible. What is the radius of convergence of the series?

3. Find the Maclaurin series for $(x^2 + 2)/(x + 3)^2$. Express your answer in sigma notation, simplified as much as possible. What is the interval of convergence of the series?

In problems 4–7, find the sum of the series. Include the open interval of convergence.

4. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n+2} x^n$

5. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{(2n)!} x^{2n}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n}}{n} x^{2n+1}$

7. $\displaystyle\sum_{n=1}^{\infty} (2n+1)(x-1)^n$

Answers:

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{2n+1} x^{4n+2}$, $-\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n (2n)!}{2^{4n+1}(n!)^2} (x-2)^n$, $R = 4/3$

3. $\dfrac{2}{9} + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (11n - 7)}{3^{n+2}} x^n$, $-3 < x < 3$

4. $-\dfrac{1}{x} - \dfrac{1}{x^2} \ln(1 - x)$, $-1 < x < 1$, $x \neq 0$; $1/2$ when $x = 0$

5. $\dfrac{x^2}{2} - \dfrac{x}{2} \sin x$, $-\infty < x < \infty$

6. $-x \ln(1 + 4x^2)$, $-1/2 < x < 1/2$

7. $\dfrac{-x^2 + 5x - 4}{(x-2)^2}$, $0 < x < 2$