

MATH 2132 Tutorial 7

In Problems 1–3, find a one-parameter family of solutions of the differential equation. Find any singular solutions.

$$1. (y-1)\frac{dy}{dx} = yx^2 \qquad 2. \frac{y-1}{y}\frac{dy}{dx} = x^2 \qquad 3. x^2\frac{dy}{dx} = y^2 - 1$$

4. Find an explicit solution of the initial-value problem

$$\frac{dy}{dx} = \frac{x^4}{y+1}, \quad y(1) = 2.$$

What is the largest possible domain of the solution?

5. Two substances A and B react to form a third substance C in such a way that 2 grams of A react with 1 gram of B to produce 3 grams of C. The rate at which C is formed is proportional to the product of the amounts of A and B present in the mixture. Find the amount of C present in the mixture as a function of time when the original amounts of A and B brought together at time $t = 0$ are 20 grams and 5 grams, respectively.

6. Find a general solution of the differential equation

$$x\frac{dy}{dx} = 3y + x^5\sqrt{1+x^2}.$$

7. Find a one-parameter family of solutions of the differential equation

$$\frac{dy}{dx} = \sin^3 x - y \sin x.$$

Are there any singular solutions?

8. When a substance such as glucose is administered intravenously into the bloodstream, it is used up by the body at a rate proportional to the amount present at that time. If it is added at a variable rate $R(t)$ units per unit time, and A_0 is the amount present when the intravenous feeding begins, find a formula for the amount present at any time. Simplify your solution when $R(t)$ is a constant value R .
9. A tank originally contains 1000 litres of water in which has been dissolved 10 kilograms of sugar. A mixture containing 2 kilograms of sugar per 100 litres of water is added to the tank at 15 millilitres per minute. At the same time, 20 millilitres of well-stirred mixture is removed from the tank each minute. Find the amount of sugar in the tank as a function of time t . For how long is your solution valid?

Answers: 1. $y - \ln|y| = x^3/3 + C$, $y = 0$ 2. $y - \ln|y| = x^3/3 + C$

3. $y = (1 + Ce^{-2/x})/(1 - Ce^{-2/x})$, $y = -1$ 4. $y = (-5 + \sqrt{10x^5 + 215})/5$, $x > -(43/2)^{1/5}$

5. $30(1 - e^{-10kt/3})/(2 - e^{-10kt/3})$ g 6. $y = Cx^3 + (x^3/3)(1 + x^2)^{3/2}$

7. $y = -(1 + \cos x)^2 + Ce^{\cos x}$

8. $A = A_0e^{-kt} + \int_0^t R(u)e^{k(u-t)}du$, $A = A_0e^{-kt} + \frac{R}{k}(1 - e^{-kt})$

9. $(10^6 - 5t)/50 - 10^{-20}(10^6 - 5t)^4$ g, $t \leq 200,000$ m