

MATH 2132 Tutorial 9

Find a general solution of the differential equation in questions 1–4.

1. $y'' + 2y' - 3y = 4e^{5x} - x$
2. $y''' + 3y'' - 4y' = 2e^x + \cos 4x$
3. $y'' + 6y' - 2y = 3x + \sin x$
4. $4y''' - 3y'' + 7y' + 2y = e^{-x/4}$
5. The roots of the auxiliary equation $\phi(m) = 0$ associated with the differential equation

$$\phi(D)y = 13x^2e^{3x} + e^x \cos 2x + 4x^3 - 5$$

are

$$m = 3, 3, 3, -1 \pm 2i, -1 \pm 2i, 3 \pm \sqrt{6}, 0, 0, -14.$$

What is the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients? Do **NOT** evaluate the coefficients.

6. (a) A 100-gram mass is suspended from a spring with constant 60 newtons per metre. It is lifted 15 centimetres above its equilibrium position and given velocity 2 metres per second upward. During its motion, it is acted on by air resistance that is equal, in newtons, to 5 times the velocity of the mass. Find the position of the mass as a function of time.
(b) What is the maximum displacement from equilibrium experienced by the mass?
(c) When, if at all, does the mass pass through its equilibrium position?

Answers:

1. $y(x) = C_1e^x + C_2e^{-3x} + (1/8)e^{5x} + x/3 + 2/9$
2. $y(x) = C_1 + C_2e^x + C_3e^{-4x} + (2x/5)e^x - (3 \cos 4x + 5 \sin 4x)/544$
3. $y(x) = C_1e^{(-3+\sqrt{11})x} + C_2e^{-(3+\sqrt{11})x} - 3x/2 - 9/2 - (2 \cos x + \sin x)/15$
4. $y(x) = C_1e^{-x/4} + e^{x/2} \left(C_2 \cos \frac{\sqrt{7}x}{2} + C_3 \sin \frac{\sqrt{7}x}{2} \right) + (4x/37)e^{-x/4}$
5. $Ax^5e^{3x} + Bx^4e^{3x} + Cx^3e^{3x} + De^x \cos 2x + Ee^x \sin 2x + Fx^5 + Gx^4 + Hx^3 + Ix^2$
- 6.(a) $(13/20)e^{-20t} - (1/2)e^{-30t}$ m (b) $(1/4)(13/15)^3$ m (c) Never