Assignment 1

1. Let V be the set of all circles in the xy-plane with centres at the origin. Include in V a circle with centre at the origin and radius zero. If C_1 and C_2 are two circles in V, define $C_1 + C_2$ to be the circle with centre at the origin with radius equal to the sum of the radii of C_1 and C_2. If a is a real scalar, define aC_1 to be the circle with centre at the origin and radius equal to $|a|$ times the radius of C_1. Is V a real vector space? Justify your answer.

2. Show that in $P_n(x)$, any set of $n+1$ polynomials, one of degree 0, one of degree 1, one of degree 2, ..., and one of degree n is linearly independent.

3. Find the rank, a basis for the row space, a basis for the column spacce, and a basis for the null space of the matrix

$$\begin{pmatrix} 1 & i & 3 & 1+i \\ 2-i & 3+i & 4 & i \\ 2i & -i & 1 & 4 \end{pmatrix}.$$

4. Prove that when W_1 and W_2 are subspaces of a vector space V, then

$$\text{dimension } (W_1 \cap W_2) + \text{dimension } (W_1 + W_2) = \text{dimension } W_1 + \text{dimension } W_2.$$

5. The first five Chebyshev polynomials are

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_2(x) = 2x^2 - 1, \quad T_3(x) = 4x^3 - 3x, \quad T_4(x) = 8x^4 - 8x^2 + 1.$$

They constitute a basis for $P_4(x)$. So also do the first five Hermite polynomials

$$h_0(x) = 1, \quad h_1(x) = 2x, \quad h_2(x) = 4x^2 - 2, \quad h_3(x) = 8x^3 - 12x, \quad h_4(x) = 16x^4 - 48x^2 + 12.$$

Find the transition matrix from the Hermite basis to the Chebyshev basis.

6. Show that the set W_1 of 2×2 symmetric matrices and the set W_2 of 2×2 skew-symmetric matrices are subspaces of $M_{2,2}(\mathcal{R})$. Verify that $M_{2,2}(\mathcal{R}) = W_1 \oplus W_2$. Find the vector component of the matrix $\begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix}$ along W_1 as determined by W_2.