Assignment 3

1. Find eigenvalues and bases for the eigenspaces of linear transformations with the following matrices:

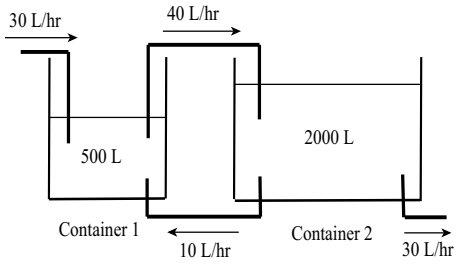
(a) $\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ -2 & 0 & 2 & -2 \\ 0 & 2 & 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -4 & -1 \\ 3 & 2 & 3 \\ 1 & 1 & 3 \end{pmatrix}$

2. A Leslie matrix is a matrix of the form

$$\begin{pmatrix} a & b & c \\ p & 0 & 0 \\ 0 & q & 0 \end{pmatrix},$$

where a, b, c, p, and q are nonnegative constants. Prove that it has exactly one positive eigenvalue.

3. Container 1 below has 500 kilograms of potassium dissolved in 500 litres of water. Container 2 has 2000 litres of pure water. Pure water is added to container 1 at 30 litres per hour, and the well-stirred mixture is pumped into container 2 at 40 litres per hour. Solution is pumped back from container 2 to container 1 at 10 litres per hour, and also removed at 30 litres per hour. Find the maximum amount of potassium ever found in container 2.



4. (a) Two masses are connected by springs as shown in Figure 4.22 of the notes. If mass M_2 is pulled 10 centimetres to the right, mass M_1 will move to the right also. Show that displacements of the masses from their equilibrium positions when both masses are stationary are

$$\frac{10k_1}{k_1 + k_2}, \qquad \frac{10k_2}{k_1 + k_2}.$$

(b) If both masses are pushed to the left from these positions with speeds of 1/2 metre per second, set up a system of initial value problems to describe their displacements $x_1(t)$ and $x_2(t)$ from equilibrium. Ignore damping, but assume that the coefficient of kinetic friction between the masses and the surfaces is $\mu = 1/20$.

(c) Use eigenvectors to decouple the system when $M_1 = 1$, $M_2 = 1/2$, $k_1 = 10$, and $k_2 = 20$, and hence find $x_1(t)$ and $x_2(t)$. Equations determining constants of integration need not be solved, but they should be shown.

(d) State in words how long your solution in part (c) is valid. You do not have to find a numerical value for the length of time, but indicate how you would find it.