MATH 3132 Tutorial 1

1. (a) Show that the following force field is conservative

$$\mathbf{F} = [3x^2ye^z + \cos(x+z)]\hat{\mathbf{i}} + x^3e^z\hat{\mathbf{j}} + [x^3ye^z + \cos(x+z) + 4z]\hat{\mathbf{k}}.$$

(b) Find all potential functions associated with the above force field.

2. Explain the diffference between a domain and a simply-connected domain.

3. Evaluate the line integral $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = 2y\hat{\mathbf{i}} + z\hat{\mathbf{j}} + 3x\hat{\mathbf{k}}$, and C is that part of the curve of intersection of the surfaces $y = x^2$ and $x + y + z = 3$ from the point $(1, 1, 1)$ to the point $(2, 4, -3)$.

4. Evaluate the line integral $\displaystyle\int_C \sqrt{x}\,dx - z\,dy + (x+y)\,dz$, where C is the curve $y - z = 1$, $x + 2 = z^2$ from the point $(2, 3, 2)$ to the point $(7, 4, 3)$.

5. Evaluate the line integral $\displaystyle\int_C (2xy - z^2)\,dx + (x^2+1)\,dy - 2xz\,dz$, where C lies in the first octant along the intersection of the surfaces $z^2 = x^2 + y^2$ and $y + z = 2$ from the point $(0, 1, 1)$ to the point $(2, 0, 2)$.

6. Evaluate the line integral

$$\int_C 64xz\,ds$$

where C is the curve $x = z^2$, $y = x + 4$ from the point $(4, 8, 2)$ to the point $(1, 5, 1)$,

7. With no calculations, explain why $\nabla \times \nabla f = \mathbf{0}$.

Answers: **1.** $-x^3ye^z + \sin(x+z) - 2z^2 + C$ **3.** -17 **4.** $(14\sqrt{7} - 4\sqrt{2} + 16)/3$ **5.** -9
6. $(1/2)\left(\frac{2\cdot 33^{5/2}}{5} - \frac{2\cdot 33^{3/2}}{3}\right) - (1/2\left(\frac{2\cdot 9^{5/2}}{5} - \frac{2\cdot 9^{3/2}}{3}\right)$