

MATH 3132 Tutorial 2

1. Evaluate

$$\int_C \left(3x^2yz + \frac{1}{x+y} \right) dx + \left(x^3z + \frac{1}{x+y} \right) dy + (x^3y + 2z) dz$$

where C is the straight line from $(1, 1, 1)$ to $(3, 4, 6)$.

2. (a) Find all domains in which the line integral

$$\int_C \frac{x dx + y dy + z dz}{x^2 + y^2 + z^2}$$

is independent of path. Justify your answer.

(b) Evaluate the line integral along that part of the curve $x^2 + y^2 + z^2 = 4$, $y = x$ in the first octant from $(\sqrt{2}, \sqrt{2}, 0)$ to $(0, 0, 2)$.

3. (a) For what value of the constant k is the line integral

$$\int_C 2xy^3 dx + (kx^2y^2 + 2y) dy - z^2 dz$$

independent of path in xyz -space?

(b) With the value of k from part (a), evaluate the line integral along the curve $y = x+1$, $z = y^2+x$ from the point $(1, 2, 5)$ to the point $(-1, 0, -1)$.

4. Evaluate the line integral

$$\oint_C (\sin x - x^2y) dx + (e^y + xy) dy$$

where C is the closed curve in the xy -plane that bounds the area enclosed by the curves $x = 2y$, $x = 2$, $y = 0$.

5. Evaluate the line integral

$$\oint_C (2xy^3 - xy) dx + (3x^2y^2 + xy^2) dy$$

where C is edge of the triangle bounded by $y = -x$, $x = -1$, $y = 0$.

6. Evaluate the line integral

$$\oint_C y^3 dx - xy^3 dy$$

where C is the edge of the triangle bounded by $x + y = 1$, $x - y = 1$, $x = 0$.

7. Evaluate the line integral $\oint_C \frac{x dx + (y-1) dy}{x^2 + (y-1)^2}$ around the curves (a) $x^2 + (y-1)^2 = 1$ and (b) $(x+1)^2 + y^2 = 1$.

8. Evaluate the line integral $\oint_C (3x^2e^y + y) dx + x^3e^y dy$, where C is the curve $x^2 + y^2 = 1$, using:

- (a) parametric equations for the curve,
- (b) independence of path,
- (c) Green's theorem,
- (d) area inside the curve.

Answers: 1. $682 + \ln(7/2)$ 2.(a) Any domain not containing the origin (b) 0 3.(a) 3 (b) 30
4. $-7/3$ 5. $-1/4$ 6. $1/2$ 7.(a) 0 (b) 0 8. $-\pi$